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ABSTRACT

This guide, published by the New York City Board of Education, presents 62 lesson plans in basic mathematics for tenth grade students. Lesson plans and performance objectives focus on the following areas: (1) fundamental operations with signed numbers; (2) linear, weight and temperature measurements; (3) fractions, decimals and percents; (4) statistics and probability; (5) perimeters, areas and volumes of geometric figures; and (6) formulas and equations. The guide has been designed to give the lessons a vocational and technical flavor, as well as to provide students in nonacademic mathematics classes with the essential skills that are required to pass the New York State Regents Competency Examination in mathematics. (APM)

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BASIC APPLIED MATHEMATICS PART 1

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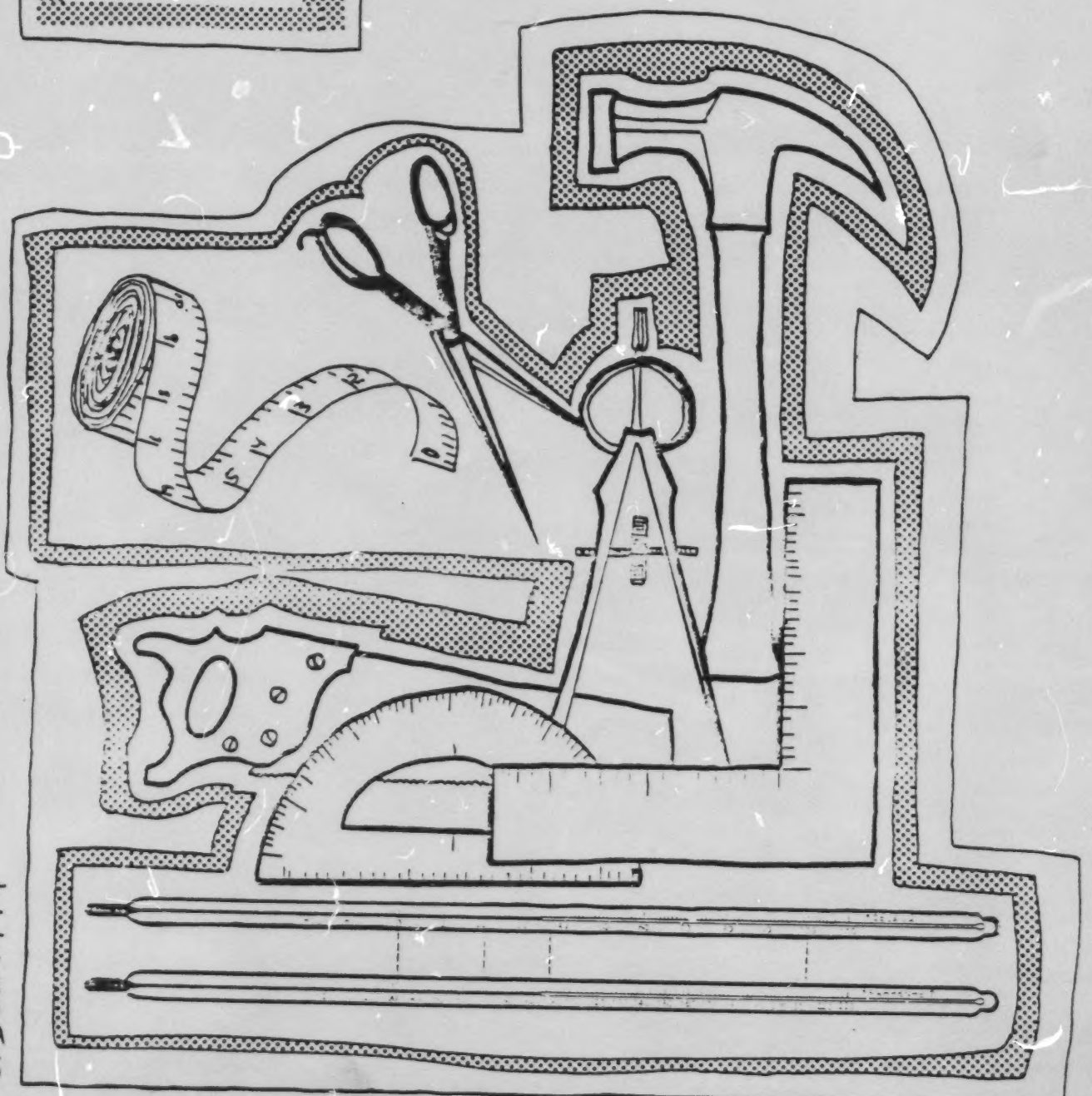

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New York City Board of Education

QUESTIONNAIRE - BASIC APPLIED MATHEMATICS - I

This is an experimental bulletin in mathematics. It needs your suggestions and support before it can be revised for maximum implementation.

1. How many classes in your school pursued this course of study?_____

2. What was the grade level of most of the students who were programmed for this course?_____

3. How were you able to coordinate the math skills with other courses of study in your school? (Woodworking, Interior Design, Plumbing, etc.)

_____

_____

4. How did the students react to the vocational flavor of this material?_____

5. How helpful was this course in helping students to pass the Regents Competency Examination in mathematics?__

_____

_____

6. Which unit or lesson stands out at the most successful in the bulletin?_____

7. What suggestions do you have for the improvement of this bulletin?

_____

_____

8. What additional comments would you make?_____

_____

_____

_____

_____

Please return this questionnaire before March 1, 1981 to:

Mrs. Maureen R. Maguire
Office of Curriculum Development and Support
131 Livingston Street - Room 617
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Foreword

This bulletin of lesson plans is the product of a felt need in our schools for a program in mathematics which relates to the world of work and yet presents real challenges to student and teacher alike.

Mathematics skills are indispensable in most fields -- carpentry, nursing, interior decorating, engineering, vehicle maintenance, plumbing, sales, horticulture, restaurant work, clothes design, and office administration. In fact, it is difficult to list occupations which do not require mathematical skills.

The problems in these lessons are arranged according to the skills involved. However, the problems address themselves to a wide variety of real work situations.

How this bulletin is used will vary from school to school. Since the motivations require a variety of measuring devices, it is (as always) up to the individual teacher to take advantage of what best suits the needs and interests of his or her students.

CHARLOTTE FRANK
Executive Director
Division of Curriculum & Instruction

ACKNOWLEDGMENTS

This experimental curriculum bulletin, Basic Applied Mathematics - Part I, was developed as a project of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

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Prologue
Performance Objectives

PERFORMANCE OBJECTIVES

CONTENT AREA AND TOPIC		REFERENCES
		All page references are to this curriculum guide, Basic Applied Mathematics, Part I.
	The student will be able to ...	
UNIT I: FUNDAMENTAL OPERATIONS WITH SIGNED NUMBERS		
LESSON 1		5 - 8
A. The Number Line	1. construct a number line using a a unit with endpoints at 0 and 1.	
	2. extend the number line to the right to include positive whole numbers.	
	3. locate any positive whole number on the number line.	
	4. compare any two numbers on the number line in terms of greater or lesser, stating that the smaller is to the left.	
B. Symbols of Equality and Inequality	1. state the meaning of the symbols $<$, $>$, and $=$.	
	2. compare two numbers on the number line by using the symbols $<$ or $>$.	
	3. state whether a given inequality is true or false.	
	4. use variables on the number line to write inequalities.	
C. Adding, Subtracting and Multiplying Positive Whole Numbers on the Number Line	1. use a number line to indicate the meaning of addition of two or more positive whole numbers.	
	2. use a number line to indicate the meaning of subtraction of two positive whole numbers where the difference is non-negative.	
	3. use a number line to indicate the meaning of multiplication of two positive whole numbers by repeated addition.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
LESSON 2		9 - 11
A. Introduction to Negative Whole Numbers	1. extend the positive whole number line to the left to include negative whole numbers.	
	2. state that the negative side of zero is to the left and the positive side of zero is to the right.	
	3. locate any given whole number on the number line.	
	4. define a negative whole number as a whole number less than zero.	
	5. define a positive whole number as a whole number greater than zero.	
B. Integers	1. define the set of integers to be the set consisting of the positive whole numbers, zero, and the negative whole numbers.	
	2. name the opposite of a given integer.	
	3. associate the relative size of two numbers with the relative positions of their points on the number line (greater number to the right, smaller number to the left).	
	4. select the greater or lesser of two integers.	
	5. arrange a given set of integers in ascending or descending order.	
C. Absolute Value of a Number	1. state the distance from zero to any number.	
	2. define movement to the left on the number line as movement in the negative direction.	
	3. write the absolute value of a negative or positive integer.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 3		12 - 14
Addition of Signed Numbers	1. define movement to the right on the number line as movement in the positive direction.	
	2. define movement to the left on the number line as movement in the negative direction.	
	3. add two positive integers on the number line.	
	4. add two negative integers on the number line	
	5. add a positive and a negative integer on the number line.	
	6. state that the sum of two positive integers is always positive.	
	7. state that the sum of two negative integers is always negative.	
	8. state that the sum of a positive integer a negative integer may be either positive or negative.	
	9. add more than two integers on the number line.	
	10. demonstrate the commutative property of addition of integers on the number line ($a + b = b + a$).	
LESSON 4		15 - 16
A. Rules for the Addition of Signed Numbers	1. state the rule for adding integers with like signs.	
	2. state the rule for adding integers with unlike signs.	
	3. compute the sum of an integer and its opposite.	
	4. compute the sum of any two integers.	
	5. compute the sum of more than two integers.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
B. Absolute Value	evaluate expressions involving the absolute values of positive and negative integers.	
LESSON 5		17 - 18
Subtracting Signed Numbers on the Number Line	1. determine the difference of two integers using the number line.	
	2. rewrite the difference of two numbers as the sum of the first number and the opposite of the second number.	
	3. find the difference of two integers in the vertical format.	
	4. find the difference of two integers in the horizontal format.	
LESSON 6		19 - 21
A. Define Factors and Product	1. express a multiplication as a series of repeated additions.	
	2. define the product to be the result of a multiplication example.	
	3. demonstrate understanding of the phrase "factors of a product" by writing a composite number as the product of two or more factors.	
	4. find the product of any number and zero.	
B. Multiplication of Signed Numbers	1. state the rules for multiplication of signed numbers.	
	2. find the product of two given signed numbers.	
	3. find the product of three signed numbers.	
	4. write an equation to illustrate the commutative property of multiplication.	
	5. write a given signed number as the product of two factors.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 7		22 - 24
Division of Signed Numbers	1. state that division by zero is not possible.	
	2. state the rules for finding the quotient of two signed numbers.	
	3. write division problems in three different ways (e.g., $\frac{15}{-3}$, $15 \div 3$, $-3 \overline{)15}$).	
	4. perform addition, subtraction, multiplication and division problems involving two or more signed numbers.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
UNIT II: LINEAR, WEIGHT AND TEMPERATURE MEASUREMENT		
LESSON 1		26 - 28
Understanding Measurement	1. provide examples of occupations in which measurements must be made (state the occupation and the substance or quantity being measured).	
	2. state that measuring a length means comparing the length to an accepted unit of measure.	
	3. use a non-standard unit of length (e.g., the length of a pencil) to measure objects in a classroom.	
	4. explain why a unit of measure must be a uniform standard.	
	5. list several ancient non-standard units of measurement.	
	6. list several modern standard units of measurement.	
	7. list some of the instruments used to measure the units above.	
LESSON 2		29 - 33
A. The Customary System of Measurement	1. identify the appropriate unit to use in measuring a given substance, object or quality.	
	2. state the relationships among the units of time: seconds, minutes, hours, days, months, years.	
	3. convert a given number of time units into another time unit (e.g., hours to days).	
B. Units of Linear Measurement	1. state the relationships among the linear units: inches, feet, yards.	
	2. convert a length expressed in terms of one linear unit into another linear unit.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	3. use a table of units of linear measurement, including miles, to answer problems requiring the conversion of units.	
C. Units of Liquid Measure	1. state the relationships among the units of liquid measure: ounces, pints, quarts, gallons.	
	2. convert a liquid measure expressed in terms of one unit into another unit.	
	3. use a table of units of liquid measure to solve verbal problems requiring the conversion of units.	
D. Units of Weight	1. state the relationships among the units of weight: ounces, pounds, tons.	
	2. convert a weight expressed in terms of one unit into another unit.	
	3. solve verbal problems requiring the conversion of units of weight.	
LESSON 3		34 - 37
A. Metric Linear Measurement	1. provide examples of occupations in this country in which metric units are used.	
	2. list the prefixes used in the metric system and their implied multiplication factors.	
	3. identify the meter as the basic unit of linear measure in the metric system.	
	4. recognize and read the millimeter markings on a ruler.	
	5. state the number of millimeters in one centimeter.	
	6. express the numerical relationships between one millimeter, one centimeter, one meter and one kilometer.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
B. Metric Weight Units	1. identify the gram and the basic unit of weight in the metric system.	
	2. estimate the weight in grams of some common objects, given the weight of objects such as a paper clip or a pair of shoes.	
	3. express the relationships among one milligram, one gram and one kilogram.	
C. Metric Units of Liquid Measure and Capacity	1. identify the liter as the basic unit of liquid capacity in the metric system.	
	2. estimate the liquid capacity of common objects or containers given the capacity of one glass or an eyedropper.	
	3. express the relationships between one milliliter, one liter and one kiloliter.	
LESSON 4		38 - 40
Operations With Units of Measure	1. express a given number of units of linear, weight or time measure in simplest form (e.g., 17 inches = 1 foot, 5 inches).	
	2. find the sum of various measurements where simplification is required.	
	3. add different measurements, where conversion is required (e.g., add 2 feet and 4 yards).	
	4. subtract various measurements where exchange is required (e.g., 6 ft 2 in. - 2 ft 9 in. = ?)	
	5. find an integral multiple of any measurement, expressed in simplest form.	
	6. divide a given measurement by a whole number from one to nine.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 5		41 - 44
Using the Customary System of Linear Measurement: the English Ruler	1. list the common units of linear measure.	
	2. draw a line on a prepared sheet visually separating an inch into two equal lengths, and label them as ½ inch.	
	3. draw lines on the prepared sheet visually separating an inch into a. fourths, and label each mark 1/4, 1/2, 3/4, b. eighths, and label each mark 1/8, 1/4, 3/8, etc., and c. sixteenths, and label each mark 1/16, 1/8, 3/16, etc.	
	4. state how many subdivisions of the inch there are in a given length of one inch or less (e.g., the number of eighths of an inch in 1/4 of an inch, quarters in one inch, etc.)	
	5. label each of the markings on pre-printed rulers of about three or four inches in length, where one ruler is divided only into halves, another is divided into quarters, another is divided into eighths and the last is divided into sixteenths.	
	6. identify the length indicated by a letter on a printed ruler.	
	7. mark a printed ruler at the point corresponding to a given length.	
	8. count the lengths (spaces) between ruler marks to determine a measurement, not the number of ruler marks.	
	9. use a 12-inch ruler to measure available objects.	
	10. use a yardstick or tape measure to measure available objects.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 6		45 - 47
Using the Metric Ruler	1. identify the yard as the unit in the customary (English) system of measurement which is closest in length to the meter.	
	2. state how many numbered subdivisions there are on a meter stick.	
	3. identify the numbered subdivisions of the meter stick as centimeters.	
	4. identify the subdivisions of the centimeter as millimeters.	
	5. express lengths in decimal notation when given in terms of centimeter-millimeter combinations, or as millimeters (e.g., 72 mm = 7.2 cm, 3 cm 6 mm = 3.6 cm).	
	6. express lengths given in terms of centimeters as a number of millimeters.	
	7. express lengths given in terms of meters as centimeters.	
	8. identify the length indicated by the letter on a printed metric ruler, using decimal notation.	
	9. mark a printed metric ruler at the point corresponding to a given length.	
	10. measure small objects using a common metric ruler (about 30 cm).	
	11. measure larger objects using a meter stick.	
LESSONS 7 - 8		48 - 55
Reading the Micrometer	1. provide examples of occupations in which measurements must be extremely accurate (to hundredths or thousandths of an inch).	
	2. measure small objects with a machinist's scale, where the inch has been subdivided into tenths.	

CONTENT AREA AND TOPIC REFERENCES

The student will be able to ...

3. identify the parts of a micrometer: anvil, spindle, hub (sleeve), thimble.

4. state that the numbered divisions on the sleeve indicate tenths of an inch.

5. read orally the number represented by any three-place decimal numeral.

6. divide a three-place decimal number by 2, 4, 5, 10 or 25 (with zero remainder).

7. express a given number of tenths of an inch (in decimal form) as the equivalent number of thousandths.

8. express a given number of hundredths of an inch (in decimal form) as the equivalent number of thousandths.

9. state that a micrometer measures thousandths of an inch.

10. state that the numbered divisions on the sleeve indicate hundred thousandths (.1 in. = .100 in.)

11. state that the numbered divisions are subdivided into four parts.

12. state that each subdivision of the numbered divisions measures .025 in. (.100 ÷ 4 = .025).

13. state that one rotation of the thimble will cause it to slide .025 in. along the sleeve.

14. state that the thimble scale has 25 marks.

15. state that each thimble scale marking indicates .001 in.

16. determine the length that the sleeve moves when it is rotated a given integral number of complete turns.

17. determine the number of thousandths of an inch that the sleeve moves when the thimble scale is turned to a given mark (mark 17 = .017 in.)

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	18. measure the dimensions of objects which are an integral multiple of .100 in.	
	19. state the length indicated by any mark showing on the hub (with the thimble scale set to zero).	
	20. measure the dimensions of objects which are an integral number of .025 in.	
	21. state the length indicated by any scale reading.	
	22. measure the dimensions of objects in thousandths of an inch which are less than one inch in length.	
LESSON 9		56 - 58
Determining the Limits of a Linear Tolerance Interval	1. state that the accuracy of a measurement is determined by the precision quality of the measuring instrument.	
	2. define tolerance.	
	3. add and subtract a given measurement expressed in inches to (or from) a given measurement expressed in feet (e.g., 150 ft + 2 in. and 150 ft - 2 in., 6 ft + .05 in. and 6 ft - .05 in.)	
	4. read a tolerance expressed with the symbol $\pm$.	
	5. define the upper limit of a tolerance interval.	
	6. define the lower limit of a tolerance interval.	
	7. determine the upper and lower limits of a tolerance interval expressed with the symbols $\pm$ or + .	
	8. determine the length of a tolerance interval.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 10		59 - 62
Reading the Fahrenheit and Celcius Temperature Scales	1. state the names of the common temeprature scales.	
	2. recognize certain significant temperatures on both the Celsius and Fahrenheit scales, such as the freezing and boiling points of water, normal body temperature, normal room temperature, etc.	
	3. read the temperatures indicated by letters on a printed thermometer scale.	
	4. indicate, on a printed thermometer scale, the points that correspond to given temperature readings expressed in tenths of a degree.	
	5. convert from one scale to the other by reading a chart showing parallel temperature scales.	
	6. use the formula $F = \frac{9}{5} C + 32$ to convert a Celsius scale reading to Fahrenheit degrees.	
	7. use the formula $C = \frac{5}{9} (F - 32)$ to convert a Fahrenheit scale reading to Celsius degrees.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
UNIT III: FRACTIONS, DECIMALS AND PERCENTS		
LESSON 1		64 - 67
What Is a Fraction?	1. divide a square, rectangle or circle into equal parts.	
	2. express verbally the relationship between a part of the figure and the whole figure (e.g., 1 out of 6).	
	3. write the verbal expression of the relationship between a part and the whole figure as a fraction.	
	4. define denominator as the number of equal parts into which the whole quantity has been divided.	
	5. define numerator as the number of equal parts represented.	
	6. rewrite a fraction as a division example.	
	7. write a relationship described in a verbal problem as a fraction.	
	8. provide examples of the use of fractions in trade areas.	
LESSON 2		68 - 70
A. Equivalent Fractions	1. define equivalent fractions.	
	2. write several fractions equivalent to a given fraction by multiplying the numerator and denominator by the same whole number.	
	3. change a fraction to an equivalent fraction with a given denominator.	
	4. choose the simplest fraction in a set of equivalent fractions.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
B. Greatest Common Factor	1. list all the factors of a given composite number.	
	2. identify two or three common factors of two given numbers.	
	3. define the greatest common factor of two numbers.	
	4. determine the greatest common factor of two given numbers.	
C. Reducing Fractions	1. change a fraction to an equivalent fraction in lower terms by dividing numerator and denominator by the same number.	
	2. state a rule for reducing fractions to lowest terms.	
	3. classify a given fraction as less than one, equal to one or greater than one.	
LESSON 3		71 - 73
A. Addition and Subtraction of Fractions With Common Denominators	1. identify fractions with common denominators.	
	2. add two or three proper fractions with common derominators (sums are less than one)	
	3. subtract fractions with common denominators.	
B. Improper Fractions and Mixed Numbers	1. write various fractions equivalent to the number one.	
	2. identify proper and improper fractions.	
	3. define a mixed number.	
	4. rewrite an improper fraction as a mixed number.	
	5. rewrite a mixed number as an improper fraction.	
	6. define perimeter.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
	7. compute a length or perimeter which requires a sum of fractions and mixed numbers.	
LESSON 4		74 - 76
A. Least Common Multiple	1. list the first six multiples of any whole number up to 20.	
	2. identify some common multiples for two given numbers.	
	3. define least common multiple.	
	4. determine the least common multiple for two or three numbers.	
B. Least Common Denominator	1. identify fractions with unlike denominators.	
	2. state the rule that to add or subtract fractions, a common denominator must be used.	
	3. state that the least common denominator is the least common multiple of the denominators.	
	4. determine the least common denominator, given two fractions with unlike denominators.	
C. Adding and Subtracting Fractions With Unlike Denominators	1. rewrite fractions as equivalent fractions with the least common denominator.	
	2. add and subtract fractions with unlike denominators.	
	3. reduce the results to lowest terms.	
	4. read a verbal problem and identify it as a problem either in addition or subtraction.	
LESSON 5		77 - 79
Adding Mixed Numbers	1. state that when adding mixed numbers the whole numbers and fractions are added separately.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	2. add mixed numbers if the fractional parts have common denominators.	
	3. add mixed numbers if the fractional parts have unlike denominators.	
	4. add a mixed number and a whole number.	
	5. add a mixed number and a fraction.	
	6. simplify the sum by:	
	a. reducing the fractional part to lowest terms,	
	b. changing improper fractions to mixed numbers, or	
	c. combining the whole number and mixed number results.	
	7. translate a verbal problem into an addition example involving mixed numbers, and solve it.	
	8. calculate the perimeters of figures by adding mixed numbers and fractions.	
LESSON 6		80 - 83
A. Subtraction of Mixed Numbers	1. find the difference of two mixed numbers when the fractions have the same denominator.	
	2. subtract two mixed numbers when exchange is not needed (the denominators of the fractions are unlike).	
B. Exchange	1. state when exchange is necessary (for fractions with common denominators in which the numerator of the first fraction is less than the numerator of the second fraction).	
	2. subtract two mixed numbers when exchange is necessary.	
	3. subtract two numbers when the first number is a positive whole number and the second number is a mixed number.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	4. subtract two numbers when the first number is a mixed number and the second number is a positive whole number.	
	5. simplify the result of subtracting mixed numbers.	
	6. calculate the dimensions of a figure by adding and subtracting mixed numbers.	
	7. calculate the dimensions of a figure by adding and subtracting mixed numbers and fractions.	
	8. translate a verbal problem into a subtraction example involving mixed numbers, and solve.	
LESSON 7		84 - 85
A. Multiplication of Fractions	1. state the rule for multiplication of fractions: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$	
	2. write an equation to illustrate the commutative property of multiplication.	
	3. multiply a proper fraction by a proper or improper fraction.	
	4. multiply an improper fraction by an improper fraction.	
B. Multiplication of Mixed Numbers	1. convert a mixed number to an improper fraction.	
	2. multiply a mixed number by a fraction or a mixed number.	
	3. multiply a whole number by a fraction or a mixed number.	
	4. express a product in the form of an improper fraction as a mixed number.	
	5. reduce a product to lowest terms.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
	6. translate the conditions of a verbal problem into a multiplication example involving mixed numbers, and solve.	
LESSON 8		86 - 87
A. Reciprocal of a Number	1. define the reciprocal of a number as that number whose product by the original number is one.	
	2. write a whole number as a fraction with a denominator of one.	
	3. write the multiplicative inverse of a number (including integers, proper and improper fractions).	
	4. state the rule for division of fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a \times c}{b \times c}$.	
B. Dividing Mixed Numbers	1. divide a fraction by a fraction (proper or improper).	
	2. divide a fraction by a mixed number or whole number.	
	3. divide a mixed number by a fraction.	
	4. divide a whole number by a fraction.	
	5. divide a mixed number by a whole number.	
	6. reduce the above quotients.	
	7. translate the conditions of a verbal problem into a division example involving mixed numbers, and solve.	
LESSON 9		88 - 90
Converting Fractions to Decimal Equivalents	1. express a fraction as an indicated division using the symbols $\div$ and $\overline{)}$.	
	2. place a decimal point after an integer dividend and in the space for the quotient of a division problem written with the symbol $\overline{)}$.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	3. add zeros after the decimal point in the dividend without changing its value.	
	4. divide a whole number divisor into a decimal number dividend.	
	5. read decimal numbers as an integral number of tenths, hundredths, thousandths, etc. (.47 = forty-seven hundredths).	
	6. write fractions and decimals as numerals, given their verbal statements.	
	7. change fractions to decimal equivalents.	
LESSON 10		91 - 92
A. Converting a Decimal to Its Fractional Equivalent	1. read basic decimals verbally (e.g., .1 = one tenth).	
	2. convert basic decimals to fractions (e.g., .01 = 1/100).	
	3. locate the decimal point when reading a mixed decimal numeral by associating it with the word "and."	
	4. write a mixed decimal numeral in fraction form (e.g., $3.875 = 3\frac{875}{1000}$).	
	5. simplify the fraction part of a mixed number.	
B. Converting Fractions and Decimals Both Ways	1. state basic decimal-fraction equivalents (e.g., $.5 = \frac{1}{2}$, $.125 = \frac{1}{8}$, etc.)	
	2. given a number written in either decimal form, fraction form or as a verbal statement, rewrite it in the other two forms.	
LESSON 11		93 - 95
Addition and Subtraction of Decimals	1. convert a decimal to a fraction whose denominator is a power of ten.	
	2. add or subtract fractions with common denominators.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	3. rewrite the resulting fraction as a decimal.	
	4. add or subtract decimals given in either a vertical or horizontal format.	
	5. state the need to line up the decimal points in addition or subtraction of decimals.	
	6. add or subtract decimals with exchange across the decimal point.	
	7. subtract a decimal number from a whole number.	
	8. translate the conditions of a verbal problem into an addition or subtraction example involving decimals, and solve.	
LESSON 12		96 - 98
Multiplying Decimals	1. write a decimal as a fraction whose denominator is a power of ten.	
	2. multiply a fraction in which the denominator is a power of ten by a whole number.	
	3. convert the resulting fraction to a decimal.	
	4. multiply a decimal by a whole number in vertical format.	
	5. determine the number of decimal places in a product by summing the numbers of decimal places in each factor.	
	6. multiply a decimal by a decimal.	
	7. translate the conditions of a verbal problem into a multiplication example involving decimals, and solve.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 13		99 - 101
Division of Decimals	1. determine that division is needed to solve a given problem.	
	2. write a division problem using the symbols $\div$ or $)\overline{\quad}$ or as a fraction.	
	3. place the decimal point in the space for the quotient in a division problem written with the symbol $)\overline{\quad}$.	
	4. translate the conditions of a verbal problem into a division example involving decimals, and solve.	
	5. multiply a decimal by a power of ten by moving the decimal point a number of places to the right.	
	6. convert a decimal divisor into a whole number by multiplying divisor and dividend by the appropriate power of ten.	
	7. divide either a decimal or whole number dividend by a decimal or whole number divisor.	
LESSON 14		102 - 104
A. Converting a Percent to a Fraction	1. write a verbal comparison of a part of a quantity to the whole quantity as a fraction.	
	2. state that "percent" means "hundredth" or "per hundred".	
	3. write a percent as a fraction having a denominator of 100.	
	4. convert percents to fractions in lowest terms.	
B. Converting a Percent to a Decimal	1. state that hundredths can be written as a two-place decimal.	
	2. write a percent as a decimal by moving the decimal point two places to the left.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
C. Converting a Decimal to a Percent	write a decimal as a percent by moving the decimal point two places to the right and adding a percent symbol.	
D. Converting a Fraction to a Percent	1. read a fraction having a denominator of 100 as a percent.	
	2. convert a fraction whose denominator is a factor of 100 to a percent by finding the equivalent fraction with a denominator of 100.	
	3. translate the verbal descriptions of comparisons between numbers into percents.	
LESSON 15		105 - 107
Finding a Percent of a Number	1. convert a percent to a fraction having a denominator of 100.	
	2. multiply a fraction with a denominator of 100 by a whole number.	
	3. convert a percent to a decimal, including such examples as 7.5% .	
	4. multiply a decimal by a whole number.	
	5. translate the conditions of a problem requiring the calculation of a percent of a number into a multiplication example involving fractions or decimals, and solve.	
	6. round off dollars and cents quantities to the nearest cent or nearest dollar.	
	7. compute the amount of sales tax on a purchase, given the amount of the sale and the percent sales tax.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 16		108 - 109
Finding What Percent One Number Is of Another	1. rewrite the verbal statement of the comparison of two numbers as a fraction.	
	2. convert a fraction to hundredths by dividing denominator into numerator or by rewriting it as an equivalent fraction with denominator 100.	
	3. write a fraction with denominator 100 as a percent.	
	4. rewrite a fraction with denominator 100 as a two-place decimal.	
	5. write a decimal as a percent.	
	6. solve a variety of verbal problems requiring the calculation of what percent one number is of another.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
UNIT IV: STATISTICS AND PROBABILITY		
LESSON 1		112 - 115
Collecting and Organizing Data	1. tally data within categories on a prepared class summary data form.	
	2. convert tally strokes to numbers.	
	3. express each tally as a fraction with the total number of data as the denominator.	
	4. convert each fraction to a percent.	
	5. check that the sum of the percents is 100%.	
	6. choose an appropriate title for a completed data sheet.	
	7. read a data summary sheet and identify the items appearing most often or least often and items receiving the highest or lowest percents.	
	8. make a judgement based upon the data (e.g., which dessert to buy for the class party).	
	9. define statistics as the science of collecting, organizing and interpreting data.	
LESSON 2		116 - 118
Selecting a Random Sample	1. record the outcomes of a random experiment (e.g., selecting cards from a well-shuffled deck).	
	2. design and complete a frequency table to summarize the outcomes.	
	3. read the frequency table to determine which event occurred most often or least often.	
	4. determine the effect upon the outcome that the number of trials has.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	5. define a random sample as a sample in which each member of the population has an equal chance of being selected.	
	6. predict the outcome of a given random experiment.	
	7. perform a variety of given random experiments, prepare a frequency table for each, and compare predicted results with experimental results.	
LESSON 3		119 - 120
Using the Mean to Interpret Data	1. compute the arithmetic average of several numbers in a given verbal problem.	
	2. define mean as the arithmetic average.	
	3. interpret the mean of a set of data as the one value which could replace all the others, if it had a frequency equal to the total number of items.	
	4. design and perform an experiment, organize the data and calculate the mean.	
	5. calculate the mean from data given in a verbal problem.	
LESSON 4		121 - 123
Using the Mode to Interpret Data	1. organize data in a given verbal problem into a frequency table.	
	2. determine, from a frequency table, the value appearing most frequently.	
	3. call the value appearing most frequently the mode.	
	4. explain that in a business the mode will indicate which item must be stocked in greater quantity.	
	5. provide examples in which a group of data may have more than one mode, or no mode.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	6. identify mean and mode as two measures of central tendency.	
	7. identify the mode as the measure of central tendency which is affected less by extreme values than the mean is.	
LESSON 5		124 - 127
Using the Median to Interpret Data	1. arrange a set of data in order of size.	
	2. explain why mean and mode are not good measures for interpreting a given set of data with widely disparate values (e.g., test scores of a class).	
	3. define the median as the measure of central tendency which is the middle number in a set of data that has been arranged in ascending or descending order.	
	4. determine which value in a set of data with an odd number of items will be chosen as the median.	
	5. find the median of a set of data with an odd number of items.	
	6. find the median of a set of data with an even number of items by taking the average of the two middle items.	
	7. determine, for a given set of data, the mode, mean and median.	
	8. determine which measure of central tendency is most appropriate in interpreting given sets of data.	
LESSON 6		128 - 129
Defining Sample Space	1. determine the possible outcomes of choice combinations systematically by using a tree diagram (e.g., cafeteria menu).	
	2. define the sample space as the set of all possible outcomes of an experiment.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	3. call each individual outcome in a sample space a simple outcome or a simple event.	
	4. determine the number of simple events in the sample space for a given three-stage experiment.	
	5. write all the simple events in the sample space of a given three-stage experiment stated verbally.	
LESSON 7		130 - 133
Probability: Basic Concepts	1. determine the number of possible outcomes of an experiment.	
	2. determine which outcome in an experiment is most likely to occur.	
	3. determine the number of times a particular event is likely to occur in a given number of trials.	
	4. express the liklihood of an outcome by comparing the number of ways in which it might happen to the total number of possible outcomes.	
	5. express the comparison of two numbers as a ratio or fraction.	
	6. define probability as: $\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$	
	7. label a probability by using the symbol P( ).	
	8. state that the probability of an event that is certain is one.	
	9. state that the probability of an event that is impossible is zero.	
	10. state that the sum of the probabilities of all the simple events in a sample space is one.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	11. calculate the probabilities of simple events in verbal problems.	
LESSON 8		134 - 136
A. Determining Probabilities Through Experiments	1. tally the outcomes of an experiment.	
	2. determine the probability of simple events based upon experimental results.	
	3. predict the probabiltiy of simple events based upon given information about an experiment.	
	4. compare the predicted results with the experimental results.	
	5. state that the greater the number of trials, the closer the experimental results come to the predicted results.	
B. Determining P (A or B)	1. define mutually exclusive events.	
	2. provide examples of mutually exclusive events.	
	3. compute the probability that either one or the other of two mutually exclusive events will occur by summing the probabilities of each: $P(A \text{ or } B) = P(A) + P(B)$.	
	4. perform experiments given in verbal problems and compare predicted results with experimental results.	
LESSON 9		137 - 140
Determining P(A and B)	1. define independent events as events where the outcome of one event will not affect the outcome of subsequent events.	
	2. compute the probability that both of two independent events will occur as the product of the probabilties of the individual events: $P(A \text{ and } B) = P(A) \times P(B)$.	

CONTENT AREA AND TOPIC

REFERENCES

The student will be able to ...

3. draw a table indicating possible outcomes of a parallel wiring circuit and a series wiring circuit.

4. determine that, for any element in a circuit, the sum of the probability that it will work and the probabiltiy that it will fail is one.

5. calculate the probabilities of the four combinations possible (e.g., A fails and B works, etc.) given hypothetical values for the probabilities that each element in a two-element circuit will work or fail.

6. solve verbal problems in which it is required to calculate the probability that two independent events will both occur.

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
UNIT V: PERIMETERS, AREAS AND VOLUMES OF GEOMETRIC FIGURES		
LESSON 1		142 - 146
Perimeter	1. determine from the conditions of a verbal problem (e.g., buying fencing material) that the solution requires the calculation of the distance around a figure.	
	2. calculate the distance around a given geometric figure by summing the lengths of its sides.	
	3. define perimeter as the distance around an enclosed figure.	
	4. calculate the perimeters of given geometric figures in which the lengths of the sides are expressed as fractions and/or mixed numbers.	
	5. calculate the perimeters of given geometric figures in which the lengths of the sides are expressed as decimals.	
LESSON 2		147 - 149
Calculating the Circumference of a Circle	1. define radius, diameter, chord and circumference.	
	2. describe the circumference of a circle as the distance around it (perimeter).	
	3. measure the circumference and diameter of a wheel or other circular object with a tape measure.	
	4. record the results of the above measurements on a chart.	
	5. calculate the ratio of circumference to diameter to two decimal places.	
	6. name the ratio of circumference to diameter pi (π).	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	7. state two approximate values of π (3.14 and 22/7).	
	8. write two formulas for circumference: $C = \pi D$ and $C = 2\pi r$.	
	9. calculate the circumference of a circle by substituting the given length of its radius or diameter into the appropriate circumference formula.	
	10. find the perimeter of a figure composed of a combination of linear and circular parts, given its dimensions in a diagram.	
LESSON 3		150 - 153
Area: Basic Concepts	1. determine from the conditions of a verbal problem (e.g., gold-plating metal rectangles) that the solution requires the measurement of surface.	
	2. determine the amount of surface in a given rectangle by counting the number of squares contained in it.	
	3. define the area of a figure as the total number of unit squares in it.	
	4. state that the area of a figure is measured in square units which correspond to the linear units used to measure its sides.	
	5. state that the number of square units in the area of a figure does not change when the squares are rearranged to form a different shape.	
	6. determine the number of square inches in one square foot, square feet in one square yard and square centimeters in one square meter, using models of each square unit.	
	7. write the symbol for square feet as ft^2, square centimeters as cm^2, etc.	
	8. generalize a formula for the area of a rectangular region ($A = lw$ or $A = bh$).	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	9. generalize a formula for the area of a square ($A = s \times s$ or $A = s^2$).	
LESSON 4		154 - 158
Calculating the Area of a Rectangle	1. write a formula for finding the area of a rectangle.	
	2. calculate the area of a rectangle by substituting its given dimensions into the formula, computing and expressing the answer.	
	3. calculate the area of an irregular figure composed of rectangles by subdividing it into separate rectangles.	
	4. write a formula for the area of a square.	
	5. calculate the areas of squares, given the lengths of their sides.	
	6. solve verbal problems requiring the calculation of the areas of rectangular, square and irregular regions.	
LESSON 5		159 - 161
Calculating the Area of a Parallelogram	1. state the formula for finding the area of a parallelogram ($A = bh$).	
	2. calculate the area of a parallelogram, given the lengths of its base and height.	
	3. state that two identical triangles, properly placed, form a parallelogram.	
	4. state that the area of a triangle is ½ the area of a parallelogram having the same base and altitude.	
	5. calculate the areas of triangles given the lengths of their bases and altitudes.	
	6. calculate the area of a triangular region in a given diagram, determining the lengths of its base and height from other dimensions given in the diagram.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 6		162 - 165
Calculating the Area of a Circle	1. state the formula for finding the area of a circle ($A = \pi r^2$).	
	2. calculate the area of a circle given the length of a radius or diameter and using 3.14 or 22/7 as an approximation to π .	
	3. calculate the areas of irregular figures composed of circular and rectangular regions in given diagrams, including the addition and subtraction of areas.	
	4. calculate the areas of irregular figures composed of circular and triangular regions in given diagrams, including the addition and subtraction of areas.	
LESSON 7		166 - 169
Volume	1. recall the definiton of area as the measure of a region.	
	2. define volume as the measure of a solid.	
	3. state that a unit of space is required to measure a solid.	
	4. provide examples of various unit solids that might be used to measure volume.	
	5. choose cubic units of volume as the most appropriate to use as standard units.	
	6. determine the volume of a solid represented in a given diagram by counting its cubic units of volume.	
	7. state that a rectangular solid (a box) has three dimensions.	
	8. provide several sets of values for the dimensions of a box that is to have a given volume.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	9. determine the volume of a rectangular solid in a given diagram that has been marked to show its cubic units by counting them.	
	10. multiply length by width by height to calculate the volume of a rectangular solid in a given diagram that has been marked to show its cubic units.	
LESSON 8		170 - 173
Calculating the Volume of a Rectangular Solid	1. state a formula for finding the volume of a box in terms of its length, width and height.	
	2. define a rectangular solid as a three-dimensional figure each of whose faces is a rectangle.	
	3. calculate the number of cubic inches in one cubic foot.	
	4. define a cube as a rectangular solid whose edges are equal in size.	
	5. write the formula for the volume of a cube ($V = e^3$).	
	6. determine the number of cubic feet in a cubic yard.	
	7. calculate the volumes of rectangular solids, given the lengths of their sides.	
	8. calculate the volumes of rectangular solids described verbally in problems.	
LESSON 9		174 - 175
Calculating the Volume of a Circular Cylinder	1. describe a cylinder as a stack of discs, each one unit in height.	
	2. state that the volume of one disc is πr^2.	
	3. state the formula for the volume of a circular cylinder. ($V = \pi r^2 h$)	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	4. state that one gallon equals 231 cubic inches.	
	5. calculate the volumes of cylinders given their dimensions verbally.	
	6. calculate the volumes of cylinders in given diagrams.	
LESSON 10		176 - 178
Calculating the Volume of a Triangular Prism	1. define a triangular prism as a solid figure whose two bases are triangles and whose sides are rectangles.	
	2. recall and list the formulas for the volumes of a rectangular solid and a cylinder.	
	3. write a formula for the volume of a triangular prism as $V = Bh$, where B is the area of the triangular base and h is the height.	
	4. calculate the volume of a triangular prism using the formula, given its dimensions in a verbal problem.	
	5. calculate the volumes of triangular prisms in given diagrams.	
	6. compare the volumes of several triangular prisms in a given diagram.	
LESSON 11		179 - 182
A. Calculating the Surface Area of a Rectangular Solid	1. define the surface area as the total of the areas of each side of a solid.	
	2. recall the formula for the area of a rectangular surface as $A = lw$.	
	3. sum the areas of each rectangular surface of a box to find the surface area.	
	4. state the formula for surface area of a rectangular solid: $S = 2(lw + lh + wh)$.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	5. state the formula for the surface area of a cube: $S = 6e^2$.	
B. Calculating the Surface Area of a Cylinder	1. provide examples of cylindrical surfaces in industrial and consumer items.	
	2. state that a cylinder is composed of two congruent circles and a rectangle.	
	3. state the formula for the surface area of a cylinder: $S = 2\pi r^2 + 2\pi rh$ or $S = 2\pi r(r + h)$.	
	4. calculate the surface areas of various cylinders given their dimensions verbally.	
LESSON 12		183 - 185
Units in Liquid Measure	1. calculate the volume of a rectangular solid.	
	2. state that one gallon = 231 cubic inches.	
	3. convert the number of cubic inches in a given volume into gallons by dividing by 231 (answer is an integer).	
	4. calculate the number of gallons in one cubic foot.	
	5. calculate the volume of a given rectangular solid in terms of gallons.	
	6. calculate the volume of a rectangular solid in cubic centimeters.	
	7. write the abbreviation for cubic centimeters as cc or cm^3.	
	8. state that 1000 cc = 1 liter.	
	9. convert a number of cubic centimeters to liters by dividing by 1000.	
	10. analyze and solve liquid measure verbal problems in standard and metric measure.	
	11. calculate volumes in terms of liquid measures in a variety of verbal problems.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
UNIT VI: FORMULAS AND EQUATIONS		
LESSON 1		188 - 190
The Order of Operations	1. identify arithmetic expressions which could have more than one answer depending on which operations are performed first.	
	2. perform calculations involving multiplications, divisions, additions and subtractions by first performing the multiplications and divisions from left to right and then performing the additions and subtractions from left to right.	
	3. analyze and solve verbal problems that require calculations to be made using the order of operations.	
	4. use parentheses as symbols of grouping.	
	5. state that operations included in parentheses are performed first.	
	6. perform calculations requiring the use of the order of operations, including parentheses.	
LESSON 2		191 - 193
Translating and Evaluating Algebraic Expressions	1. differentiate between complete English sentences and phrases (expressions).	
	2. differentiate between mathematical expressions and sentences.	
	3. define variable as a letter which is used to represent a number.	
	4. use variables, numerals and the symbols $<$, $>$ and = to write algebraic sentences from verbal statements.	
	5. evaluate algebraic expressions by substituting given numerical values for the variables.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	6. construct and complete a table of values to calculate and list values of a given algebraic expression for given values of the variable.	
LESSON 3		194 - 196
Evaluating Algebraic Expressions Involving Powers	1. recall the definitions of product, factor and repeated factor.	
	2. write the product of repeated factors as a base with an exponent.	
	3. read expressions such as 6^2, 5^3, etc. as squared, five cubed," etc.	
	4. identify the parts of expressions such as x^4 as the base, exponent and power.	
	5. write the exponent of a number which is not a repeated factor as 1.	
	6. calculate the value of an algebraic expression involving powers, given the values of the variables.	
LESSON 4		197 - 198
Writing a Formula From a Verbal Sentence	1. find the missing value(s) in a table of values of three variables in which the value of one variable is the product of the other two.	
	2. express the above relationship as a mathematical rule, stated verbally.	
	3. rewrite the rule above as a formula using variables.	
	4. translate a variety of verbally stated relationships between variables into algebraic formulas.	
	5. translate a variety of algebraic formulas into English sentences.	

CONTENT AREA AND TOPIC	The student will be able to ...	REFERENCES
LESSON 5		199 - 200
Evaluating Commonly Used Formulas	1. write the formula for finding the surface area of a cylinder.	
	2. calculate the surface area of a cylinder, given its dimensions in a verbal problem.	
	3. prepare a list of commonly used formulas within specific trade areas.	
	4. calculate the solutions to verbal problems by evaluating formulas, given the values of the variables.	
LESSON 6		201 - 202
Solving Algebraic Equations of the Form $x \pm a = b$	1. translate a verbal sentence into an algebraic sentence with a variable.	
	2. define an open sentence as a sentence with a variable in it.	
	3. define an equation as a mathematical sentence which states that two quantities are equal.	
	4. define solution as a value of a variable that makes an equation true.	
	5. solve an equation of the form $x \pm a = b$ by adding the opposite to both sides of the equation.	
	6. use the principle of substitution to check the solution of an equation.	
LESSON 7		203 - 205
Solving Equations of the Form $ax = b$ and $\frac{x}{a} = b$	1. solve a verbal problem by writing a formula of the form $ax = b$, substituting a given value of b and determining a by trial and error.	
	2. state the division property of equations.	
	3. solve an open sentence of the form $ax = b$ by dividing both sides by a.	

CONTENT AREA AND TOPIC		REFERENCES
	The student will be able to ...	
	4. solve an open sentence of the form $\frac{x}{a} = b$ by multiplying both sides of the equation by a.	
	5. solve and check equations of the form $\frac{ax}{c} = b$ by using multiplication and division properties and substitution.	
LESSON 8		206 - 207
Solving Equations of the Form $ax \pm b = c$	1. translate the conditions of a verbal problem into a mathematical sentence of the form ax b = c.	
	2. solve equations of the form $ax \pm b = c$ by dividing first.	
	3. solve equations of the form $ax \pm b = c$ by adding first.	
	4. choose the method of adding first as the most convenient.	
	5. state a rule for solving equations of the form $ax \pm b = c$.	
	6. solve and check a variety of equations of the form $ax \pm b = c$.	

Introduction

This set of sixty-two lesson plans has been prepared primarily for use by tenth-year general mathematics classes in the vocational and technical high schools. As an experimental curriculum bulletin, it was designed not only to give the lessons a vocational and technical flavor, but also to provide students in non-academic mathematics classes with the essential basic minimum skills that are required to pass the New York State Regents Competency Examination in Mathematics. The lessons have been developed in six units. In some classes, the teacher may find it desirable to extend or omit selected lessons according to the needs and interests of the students, attention spans, attendance patterns, and time schedules.

The selection of units includes skills in algebra and geometry, statistics and probability, and metric and customary measurements. References to the August 1978 N.Y.S. Education Dept. bulletin, *General High School Mathematics, are indicated at the beginning of each unit. Most of the units are independent of each other, thus allowing students who have been absent for extended periods of time or who have erratic attendance patterns to successfully participate in class activities. An ample number of suggested exercises has been prepared to give the teacher an excellent source of applications. Students who have mathematical deficiences in basic skills, concepts, and problem solving are given the opportunity to constantly review and reinforce the material being taught. A set of performance objectives to correspond to each lesson has been included as a "Prologue" to the bulletin.

Teachers are encouraged to experiment with sequences other than those indicated for each of the units, according to the needs of their students. The lessons and the performance objectives are only a guide to encourage teachers to develop problems and exercises to supplement the units. The developmental format of the lesson plans has been used not only to help teachers organize and teach their lesson, but also to encourage students to get involved in the thinking processes so necessary for the improvement of basic mathematics skills. Suggestions for changes, additions, deletions, and (of course) improvements in this guide are cordially welcomed. (See accompanying questionnaire.)

Unit I

FUNDAMENTAL OPERATIONS WITH SIGNED NUMBERS

Lesson 1 To introduce the number line and illustrate its use in arithmetic operations
(GHSM* pp. 11-12)

Lesson 2 To extend the number line to include the set of integers
(GHSM pp. 11-12)

Lesson 3 To add signed numbers using the number line
(GHSM pp. 11-12)

Lesson 4 To add signed numbers using absolute values

Lesson 5 To subtract signed numbers, using the definition of subtraction
(GHSM pp. 11-12)

Lesson 6 To multiply signed numbers
(GHSM pp. 17-18)

Lesson 7 To divide signed numbers
(GHSM P. 18)

*General High School Mathematics, New York State Education Department, August 1978.

UNIT I

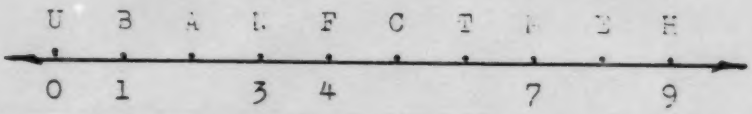
FUNDAMENTAL OPERATIONS WITH SIGNED NUMBERS

Lesson 1

Aim

To introduce the number line and illustrate its use in arithmetic operations

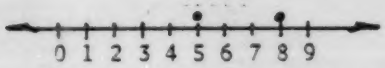
Motivation



Insert the missing numbers on the number line above and decode the following message:

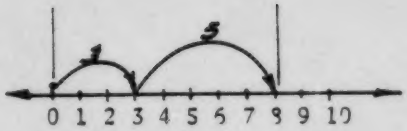
7 2 6 9 5 2 3 1 8 4 0 3

Development

Teacher Activity	Student Activity
1. Point out the one-to-one correspondence between the numbers and the points.	1. The message is "math can be fun."
2. The ordering property can be established using a number line. Which is greater, 5 or 8? On the number line, where are the points associated with 5 and 8 ? How can we use the number line to determine the greater of two numbers?	2.  8 > 5 The point associated with 8 is farther to the right than the point associated with 5. The greater number is associated with a point farther to the right than a lesser number.
3. Which of the following statements are true? a. $3 > 7$ b. $4 < 9$ c. $4 < 1$ d. $0 > 5$ e. $0 < 7$	3. a. False b. True c. False d. False e. True

4. Have the students draw a number line from 0 to 10 and ask them to find the sum of 3 and 5.
Start from zero.
When adding, move to the right.

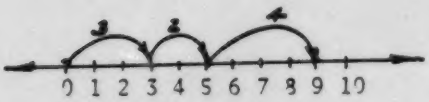
4. Start



Addends: 3
5
Sum: 8

5. Have the students use the number line to add 3 + 2 + 4.

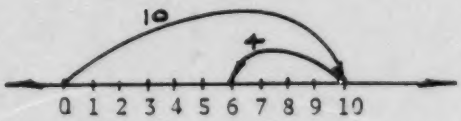
5.



Addends: 3
2
4
Sum: 9

6. Have the students use the number line to subtract 4 from 10.
When subtracting, move to the left.

6.



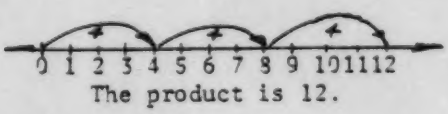
Difference = 6

7. Have the students use the number line to multiply **3 times 4.**

Elicit from the students that multiplication is repetitive addition.

3 x 4 = 4 + 4 + 4

7.



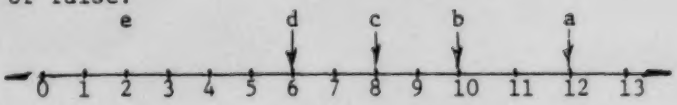
The product is 12.

Summary

1. When comparing numbers on a number line, how can we determine which is greater?
2. How can we find the sum of 4 and 3 on the number line? Explain.
3. Explain how you can subtract 7 from 12 on the number line.
4. How can you use the number line to find the product of 5 and 2?

SUGGESTED EXERCISES

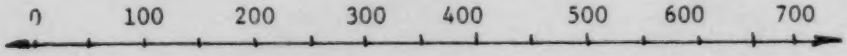
1. Using the number line to the right, indicate whether each of the following statements is true or false.



a) $a > b$ b) $d > c$ c) $b > d$ d) $d < 2$

e) $4 > a$ f) $e = 2$ g) $c = 2$ h) $a = 3$

2. Using the number line below, find the sum of each of the following:



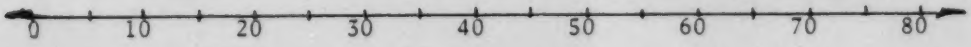
a) 100 + 300

b) 150 + 300

c) 150 + 50 + 400

d) 250 + 100 + 350

3. Using the number line below, find the difference for each of the following examples:

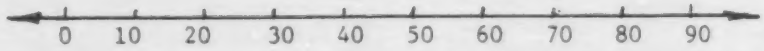


	a	b	c	d	e	f	g	h
minuend	80	75	30	65	65	20	35	70
subtrahend	70	15	25	25	10	15	35	0
difference								

4. Using the number line below find the <u>product</u> of each of the following examples:

a) 3 x 20

b) 4 x 10



c) (2) (40)

d) (3) (15)

e) 3 x 25

(Note to teacher: Discuss the different symbols representing multiplication.)

5. Indicate whether each of the following statements is true or false.

a) $x > y$ b) $y < 7$

c) $b > x$ d) $a < b$

e) $x > 7$

Lesson 2

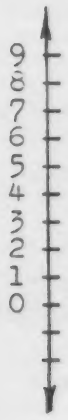
Aim

To extend the number line to include the set of negative whole numbers.

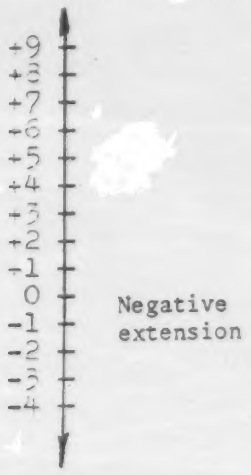
Motivation

The temperature in the morning was 4° C. By noon it had increased 5°. Use the vertical number line to find the temperature at noon.

If the temperature in the morning was 4° C, and it fell 5° during the day, can this same number line be used to determine the new temperature? Why?



Development

Teacher Activity	Student Activity
1. Have students assign values to extend the number line below zero. Since there will be two points, each of which is one unit from zero, but in opposite directions, we must distinguish between them. A number less than zero is a negative number and is indicated by the symbol "-". A number greater than zero is a positive number and is indicated by the symbol "+". Our number line is now divided into three parts: the positive whole numbers, zero, and the negative whole numbers. This set of numbers is called the set of integers.	1. At noon the temperature was 9° C. If the temperature fell 5° it would be below zero. 
2. Opposites are numbers which are the same distance from zero, but in opposite directions. The opposite of -5 is +5. The opposite of 13 is -13.	

3. a) How can the greater of two numbers be determined on a horizontal number line?

   b) How can the greater of two numbers be determined on a vertical number line?

3. a) The one that is farther to the right is greater.

   b) The number that is above the other is the greater.

4. Using a number line, determine which number is greater.

   a) +2 or -1
   b) -2 or 0
   c) 0 or -3
   d) -3 or +2

4.

   a) +2 > -1
   b) 0 > -2
   c) 0 > -3
   d) +2 > -3

5. Inform students that the distance from zero to a number is known as the absolute value or magnitude of the number.

   The symbol | | is used to indicate the absolute value of a number.

   $|-3| = |3|$

   $|3| = |3|$

Summary

1. What was done to the number line to include negative numbers?

2. How can the greater of two numbers be determined on (a) a horizontal number line, (b) a vertical number line?

3. What is meant by the absolute value or magnitude of a number?

4. What symbol is used to indicate absolute value ?

Suggested Exercises

1. Find the opposite of each of the following:

   a) -7 b) $+5$ c) -6 d) $-1\frac{2}{3}$ e) $+7.5$ f) 0

2. Arrange the following numbers in descending order (greatest first).

   a) $+3, +8, -15, -30, -7, +14, 0$

   b) $-15, -14, -12, 15, 19, 75, 28$

3. Find the value of each of the following:

   a) $|-3| =$ b) $|+9| =$ c) $|-50| =$ d) $|0| =$

4. Arrange the following numbers in ascending order (least first).

   a) $-5, -11, +15, +2, 0, -28, -1, +5, +9$

   b) $0, 1, 2, 3, -3, -4, -5, -8, -12, 7, 4$

Lesson 3

Aim

To add signed numbers using the number line

Motivation

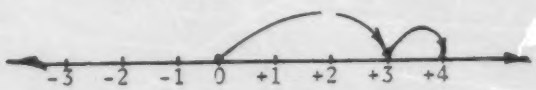
In one week, the following daily changes took place in the price of a company's stock: +3, +1, -2, +2, -4.

Use a number line to find the total change in the price of the stock during the week.

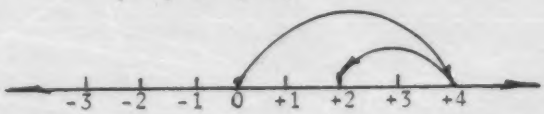
Development

Teacher Activity	Student Activity
1. a) Have students realize that the total change can be found by adding the five changes on the number line.	
b) Develop a convention for the addition of signed numbers on a number line.	
Moving to the <u>right</u> will be the <u>positive</u> direction.	
Moving to the <u>left</u> will be the <u>negative</u> direction.	

(+3) + (+1) =



(+4) + (-2) =



2. Add:

a) (+7) + (+3)
b) (+3) + (+7)
c) (-4) + (-5)
d) (-5) + (-4)
e) (+9) + (-3)
f) (-3) + (+9)

2.

a) +10
b) +10
c) -9
d) -9
e) +6
f) +6

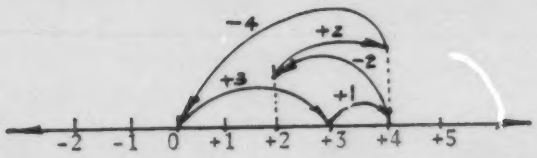
3. Elicit that changing the order of the addends does not affect the sum.

This is called the commutative property for addition.

We represent it: $a + b = b + a$.

4. Have the students find the total change in the motivation problem.

   Conclude that the price of the stock had a total change of 0; it neither increased nor decreased in value for the week.



5. Have the students add the same numbers, but in a different order.

5. The sum is always 0, no matter what the order.

Summary

1. What is the result of adding +6 and +2? Will the sum of two positive numbers always be positive? Explain.

2. What is the result of adding -6 and -2? Will the sum of two negative numbers always be negative? Explain.

3. What is the sum of +6 and -2? What is the sum of -6 and +2? When a positive number and a negative number are added, can we determine the sign of the sum? Explain.

Suggested Exercises

1. Use the number line to add the following:

	a	b	c	d	e	f	g	h	i	
addend	+9	-6	-3	-9	-4	-5	+8	-7	-5	+7
addend	-8	+4	-9	-5	+11	+2	-2	-4	+11	-7
Sum										

2. Find each of the following sums, using the number line.

a) (-5) + (+2) b) (+5) + (+2) c) (+5) + (-2) d) (-8) + (+3)

e) (-6) + (-10) f) (-4) + (-3) + (-7) g) (-8) + (-3) + (+2) + (-1)

h) (+7) + (-9) + (-3) + (+10)

3. Using a number line, find the sums of these problems. (Let each unit = 100.)

a)	b)	c)	d)	e)
+750	-800	+750	-350	-200
-250	-250	-200	-150	+300
		-100	+500	+150

4. Find the sum of each of the following examples, using a number line.

a)	b)	c)	d)	e)	f)
+8	-2	-2	-7	-3	-9
-2	-3	+2	+8	-4	-2
+6	-5	-3	-10	-2	-3
-2	-9	+3	+3	-9	-7
				-1	+21
				+12	

Lesson 4

Aim

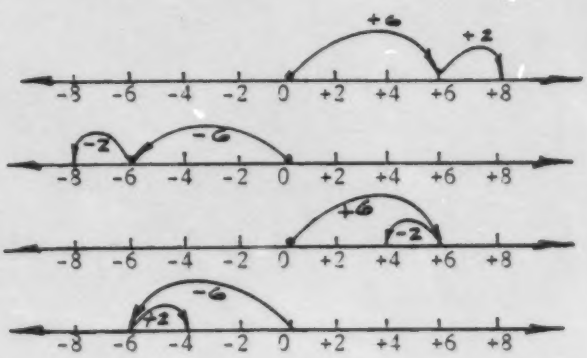
To add signed numbers using absolute values

Motivation

Use a number line to add each of the following:

a) $\begin{array}{r} +6 \\ \underline{+2} \end{array}$ b) $\begin{array}{r} -6 \\ \underline{-2} \end{array}$ c) $\begin{array}{r} +6 \\ \underline{-2} \end{array}$ d) $\begin{array}{r} -6 \\ \underline{+2} \end{array}$

Development

Teacher Activity	Student Activity
1. Using the sums obtained, and recalling the responses to the summary of lesson 3, elicit these rules: Rule 1 - When adding numbers with like signs, find the sum of their absolute values, and use the common sign. **Rule 2 - When adding two numbers with unlike signs,** find the difference of their absolute values, and use the sign of the number with the greater absolute value.	1. 
2. Have students find these sums:	2.
a) $\begin{array}{r} -15 \\ \underline{+10} \end{array}$ b) $\begin{array}{r} -9 \\ \underline{-9} \end{array}$ c) $\begin{array}{r} +6 \\ \underline{-4} \end{array}$ d) $\begin{array}{r} -8 \\ \underline{+8} \end{array}$	a) -5 d) 0 b) -18 e) -16 c) +2 f) +3
e) (+7) + (+9) f) (-5) + (+3)	
3. When two opposites are added, what is their sum?	3. The sum of a number and its opposite is zero.

Summary

1. What is the rule for adding numbers with like signs?
2. What is the rule for adding numbers with unlike signs.
3. What is the sum of a number and its opposite?

Suggested Exercise

1. Find the sum for each of the following:

a	b	c	d	e	f	g	h	i
+10	-6	-8	-9	-5	-7	+9	-7	-9
- 8	+2	-11	-15	+12	+3	-5	-13	+9

2. Add. Verify the result using the number line.

a) (-6) + (-3) b) (+14) + (-11) c) (-9) + (-15)

d) (-7) + (+5) e) (-6) + (+7) f) (-20) + (+20)

g) (+20) + (+20) h) (-20) + (-20) i) (-75) + (-15)

3. Find the sum:

a) -780 b) -850 c) +1000 d) -1500
+340 -150 - 200 - 300

e) -1875 f) +5280 g) -615 h) -775
- 925 +5957 +915 -325

4. Add:

a) -8 b) -8 c) -7 d) -7 e) -5 f) -5
-9 +9 +6 -6 -7 -9
-3 +3 +11 +3 -2 +4
-1 +10

g) +857 h) -867 i) -905 j) +1150 k) -6250
+132 -932 -603 - 275 +8760
+765 -145 +452 -593 -5775

5. Find the value of each of the following:

a) $|+5|$ b) $|-10|$ c) $|-52|$ d) $|+75|$ e) $|0|$

6. Indicate whether each of the following statements is true or false.

a) $|-5| = 5$

b) $|+8| = |-8|$

c) $(8) + (-7) = (-7) + (-8)$

d) $|-8| + |-7| = |(-7) + (-8)|$

e) $|-8| + |+8| = 0$

f) $(-8) + (+8) = 0$

Lesson 5

Aim

To subtract signed numbers, using the definition of subtraction

Motivation

At 3 p.m. last Tuesday, the temperature was $+5^{\circ}$ F. By 5 a.m. the next morning, the temperature had dropped to -7° F. Using a number line, determine the difference in temperature.

Development

Teacher Activity	Student Activity
1. Discuss the problem and elicit that the difference in temperature is a drop of 12°, or a change of -12°.	
2. Have the students write the example in vertical format. Note: The earlier reading is always subtracted from the later reading. Thus, a drop from 8° to 3° is computed: $3 - 8 = -5$.)	2. Subtract -7 (final reading) $+5$ (earlier reading) -12 (difference)
3. Have students do this example alongside the subtraction example, and compare them. Add: -7 -5	3. Subtract: -7, $+5$, -12 ← opposites → Add: -7, -5, -12
4. Conclude that subtracting a given number is equivalent to adding its opposite.	4. $a - b = a + (-b)$
5. Rule: To subtract signed numbers, change the sign of the subtrahend, and add.	5. Subtract: -2, -6 becomes Add: -2, $+6$
6. Illustrate both the vertical and horizontal format.	6. $(+5) - (+3)$ becomes $(+5) + (-3)$.

Summary

1. How can we subtract -9 from a number?
2. How can we subtract $+17$ from a number?
3. What is the rule for subtracting signed numbers?

Suggested exercises

1. Subtract:

	a	b	c	d	e	f	g	h	i	j	k	l	m
minuend	+5	-8	-8	-8	-7	-7	+7	0	+3	-15	-15	-15	12
subtrahend	-2	+3	-4	-12	+7	-7	-7	+3	0	+3	+13	-13	-9
Difference													

2. Perform the indicated operations:

a) (+9)-(-3)
b) (+15)-(+24)
c) (-28)-(-9)
d) (-14)-(+50)
e) (-15)-(75)
f) From -200, subtract -1000
g) From -1000, subtract -200
h) Subtract -250 from +750
i) Find the change in temperature during the day: temperature 8 a.m. - 20°, temperature 3 p.m. $+10^{\circ}$

3. Perform the indicated operations:

a) (-6)-(9)
b) (-6)+(9)
c) (+5)+(-5)
d) (+5)-(-5)
e) (+50)+(-20)
f) (-45)-(55)
g) (-275)-(+100)
h) (+100)-(-275)
i) (+100)+(-275)
j) (+50)+(-40)-(-60)

4. Add the following:

a) +8, -2
b) -9, -3
c) -7, +5
d) -6, +15
e) +18, +14
f) -7, -9

5. Subtract the following:

a) +8, -2
b) -9, -3
c) -7, +5
d) -6, +15
e) +18, +14
f) +7, +9

6. Perform the indicated operations:

a) (+16) + (-11)
b) (-8) + (-5)
c) (-8) - (-5)
d) (+24) - (+36)
e) (+24) + (+36)
f) (+36) + (+24)
g) (+36) - (+24)

Lesson 6

Aim

To multiply signed numbers

Motivation

On a cold day last winter, the temperature dropped three degrees per hour for four consecutive hours. What was the total change in temperature during that four-hour interval?

Development

Teacher Activity	Student Activity
1. Have students show the change on the number line. Have them record the change in both vertical and horizontal format.	1. $(-3) + (-3) + (-3) + (-3) = -12$ Add -3 -3 -3 -3 -12
2. Can this problem be worked out another way, without using addition?	2. We can use multiplication, because the same number (-3) is being added repeatedly.
3. How can we rewrite the addition?	3. $(-3)+(-3)+(-3)+(-3) = 4 \times (-3)$ So, $4 \times (-3) = -12$
4. What do we call the result or answer obtained when we multiply?	4. The result is called the product.
5. What do we call the numbers multiplied to obtain a product?	5. The numbers multiplied are called factors.
6. If four groups of (-3) are equal to -12, what will three groups of (-3) equal? What will two groups equal?	6. $3 \times (-3) = -9$ $2 \times (-3) = -6$
7. When a positive number is multiplied by a negative number, what is the sign of the product?	7. The product of a positive number and a negative number is a negative number.
8. Using the products already found, have the students develop a chart similar to the one shown below. Have the students examine and discuss the pattern developing, particularly in the column of products.	

9. a) Recall that:
(1) x (a) = a
(0) x (a) = 0

b) Elicit that, as the first factor decreases by one, the product increases by three.

c) Using this pattern, have the students complete the product column.

9.

First Factor		Second Factor		Product
(+4)	x	(-3)	=	-12
(+3)	x	(-3)	=	-9
(+2)	x	(-3)	=	-6
(+1)	x	(-3)	=	-3
(0)	x	(-3)	=	0
(-1)	x	(-3)	=	
(-2)	x	(-3)	=	
(-3)	x	(-3)	=	
(-4)	x	(-3)	=	

10. Based upon the chart, what is the sign of the product when we multiply two negative integers?

10. The product of two negative integers is positive.

11. By referring back to the number line, elicit the rule for the multiplication of two positive integers.

11. The product of two positive integers is positive.

12. Find these products:

a) (-3) x (+4)
b) (+4) x (-3)
c) (-5) x (-2)
d) (-2) x (-5)
e) (+6) x (+9)
f) (+9) x (+6)

12.

a) -12
b) -12
c) +10
d) +10
e) +54
f) +54

13. Elicit that changing the order of the factors does not affect the product. This is the commutative property of multiplication.

13. a x b = b x a

Summary:

1. When we multiply (+5) by (+4), what are the factors? What is the product?

2. When we multiply two positive integers, what is the sign of the product?

3. When we multiply a positive integer by a negative integer, what is the sign of the product?

4. When we multiply two negative integers, what is the sign of the product?

5. What is meant by the commutative property of multiplication?

Suggested Exercises

1. Find the products of each of the following:

a) (+3)(+6)
b) (+3)(-6)
c) (-6)(+3)
d) (-6)(-3)
e) (-3)(-6)
f) (-7)(-4)
g) (+15)(+4)
h) (+15)(-2)
i) (-10)(0)
j) (-25)(-12)
k) (+55)(-18)
l) (+625)(-45)
m) (-825)(-324)

2. Multiply:

a) (+6)(+3)
b) (-5)(-4)
c) (-2)(+7)
d) (8)(-10)
e) (12)(-3)
f) (-1)(-7)
g) (-4)(4)
h) (6)(-2)

3. Fill in each parenthesis with the appropriate value.

a) (+2)( ) = +6
b) (+4)( ) = -8
c) (-3)( ) = -18
d) (-4)( ) = +12

4. Perform the indicated operations.

a) (+6)(-2)
b) (+6)-(-2)
c) (+6)-(-2)
d) (+15)+(-2)+(-9)
e) (+6)-(-2)
f) (-25)+(-2)(-10)
g) (-152)(29)
h) ((+785)+(-952)
i) (+452)-(-128)
j) (-875)+(-215)
k) (-1239)(+875)
l) (-1239)-(+875)

Lesson 7

Aim

To divide signed numbers

Motivation

On a cold night last winter, the temperature fell 20° over a four-hour period of time. How can we calculate the average change in temperature in degrees per hour?

Development

Teacher Activity	Student Activity
1. What integer represents a drop of 20 degrees?	1. -20
2. Into how many equal parts must the -20 be divided?	2. Into four equal parts: $-20 = (-5)+(-5)+(-5)+(-5)$
3. Elicit that the average hourly change was -5 degrees.	3. Therefore, $\frac{-20}{4} = -5$
4. Since 4 can be written as +4, $\frac{-20}{+4} = -5$. What rule does this suggest for dividing a negative integer by a positive integer?	4. since $-20 = (+4)(-5)$ The quotient of a negative integer and a positive integer is negative.
5. Recalling that multiplication is the inverse operation of division, have students express each of these as multiplication examples, and solve: a) $\frac{-8}{-4}$ b) $\frac{+10}{-2}$ c) $\frac{+6}{+2}$	5. a) $\frac{-8}{-4} = ?$ $(-4) \times (\ ? \ ) = -8$ $(-4) \times (+2) = -8$ Therefore, $\frac{-8}{-4} = +2$ b) $\frac{+10}{-2} = -5$ c) $\frac{+6}{+2} = +3$
6. From these examples, develop the rules for the division of two signed numbers.	6. The quotient of two positive integers is positive. The quotient of two negative integers is positive. The quotient of a positive integer and a negative integer is negative.

7. Using division as the inverse operation of multiplication, have students note that division by zero is not possible.

7. $\frac{8}{0} = ?$

$(0) \times (\ ?\ ) = 8$

There is no number which, when multiplied by zero, gives a product of eight.

Summary

1. When +6 is divided by +3, what is the quotient? What is the divisor?

2. What is the sign of the quotient when we divide two positive integers?

3. What is the sign of the quotient when we divide two negative integers?

4. What is the sign of the quotient when we divide a positive integer and a negative integer?

5. Are the rules for multiplying integers similar to the rules for dividing integers? Explain.

6. Which number is never permitted as a divisor?

Suggested Exercises

1. Find each of the following quotients:

a) $\frac{-12}{-2}$ b) $\frac{+15}{-3}$ c) $(-18)\div(-3)$ d) $(+21)\div(-3)$

e) $\frac{-144}{+4}$ f) $\frac{+125}{-5}$ g) $\frac{-81}{-27}$ h) $(-72)\div(-9)$

i) $35\overline{)-420}$ j) $-10\overline{)-100}$

2. Perform the indicated multiplications:

a) $(-9)(-2)$ b) $(-30)(+4)$ c) $(-12)(-3)$

d) $(-7)(-3)$ e) $(-35)(+15)$ f) $(-8)(-3)$

3. Perform the indicated operations:

a) $(-5) + (-3) + (-2) + (-11)$

b) $(-28) - (-5)$

c) $(+20) + (-5) + (-6) + (-9) - (-20)$

d) $(-15)(4)$

e) $(-20)(-10) \div (-5)$

f) $(-6.4)(-3.2)(-5.9)$

g) $(-2528) + (-2719) + (+872) + (-7523)$

h) $(-75) + (-150)$

i) $(-35)^2$

j) $(-7)(-2) + (-33) \div (-11)$

4. Perform the indicated divisions:

a) $\frac{-12}{-2}$ b) $\frac{+15}{-3}$ c) $(-18) \div (-3)$ d) $(+21) \div (-3)$

e) $\frac{-144}{+4}$ f) $\frac{+125}{-5}$ g) $\frac{-81}{-27}$ h) $(-72) \div (-9)$ i) $35\overline{)-420}$

Unit II

LINEAR, WEIGHT, AND TEMPERATURE MEASUREMENTS

Lesson 1	To develop an understanding of measurement (GHSM p. 39)*
Lesson 2	To introduce the Customary System of measurement
Lesson 3	To introduce the metric system of measurement (GHSM p. 39)*
Lesson 4	To perform the fundamental operations with units of measure
Lesson 5	To develop skills in the Customary System of linear measurement
Lesson 6	To develop skills in the use of the metric ruler (GHSM p. 39)*
**Lesson 7	To read the micrometer
**Lesson 8	To develop skill in the use of the micrometer
Lesson 9	To determine the upper and lower limits of a tolerance interval
Lesson 10	To measure temperature in the Fahrenheit and Celsius scales

*General High School Mathematics, New York State Education Department, August 1978.

**Delete in schools where the use of the micrometer is not pertinent to the curriculum.

UNIT II

LINEAR, WEIGHT AND TEMPERATURE MEASUREMENTS

Lesson 1

Aim

To develop an understanding of measurement

Motivation

In almost every kind of work some measuring must be done. Name some occupations in which accurate measurements are necessary.

Materials

Ruler, thermometer, clock, micrometer, protractor, compass.

Development

Teacher Activity	Student Activity
1. Elicit answers to the motivation problem.	1. Some possible responses: a) electrician — measures length and diameter of wire; reads meters b) nurse — measures volume, temperature, time and weight c) navigator — measures angles and distances d) beautician — measures volume and temperature of liquids e) carpenter — measures dimensions of boards, etc f) machinist — measures dimensions of metal parts
2. a) Pose the question: What are we actually doing when we make a measurement?	2. a) When we measure an object we compare it with some unit of measure.
b) Have a student demonstrate the answer by using a pencil to measure the width of a desk.	b) The student will determine that the desk is a certain number of 'pencils' wide.
c) Does a pencil make a good measuring instrument or unit of measure?	c) No, because pencils vary greatly in size.

3. a) Discuss the fact that, for many centuries, mankind relied upon non-standard units of measure. Have the students name some of these common non-standard units.

3. a) digit - the width of a finger
palm - the width of a hand
span - the width of a hand with fingers extended
cubit - distance from elbow to fingertips
yard - distance from nose to fingertips with arm outstretched
fathom - distance from fingertip to fingertip with arms extended
foot - length of a person's foot
mile - 1000 paces
a pinch, a dash, a drop, etc

b) Have the students realize that such non-standard units are not adequate for modern life, with skyscrapers and bridges, tape recorders and tiny watches, miracle drugs and jets crossing the ocean.

c) Inform students that a modern automobile has over 200 points of contact that require measurements with a permissible error of less that five ten-thousandths of an inch.

4. a) Elicit that all measurements we will use must have a uniform value. We call these uniform values standard units of measure.

Have the students list some of these standard units.

4. a) Time: day, hour, minute, second.
Weight: ton, pound, ounce.
Length: mile, yard, foot, inch.
Liquid: pint, quart, gallon.
Angles: degrees.
Metric weight: kilogram, gram.
Metric length: kilometer, meters, millimeter.
Metric capacity: liter.

b) What are some of the instruments used to measure these units in various occupations?

Demonstrate these instruments, as available.

b) The ruler, yardstick, meterstick, micrometer, thermometer, clock, scale, measuring cups, meters, sextant, protractor.

5. Tell students that the Bureau of Standards in Washington, D.C. has the final authority to define the standard units for the United States.

Summary

1. Name some non-standard units of measure.
2. Name some standard units of measure.
3. When we measure an object, what are we actually doing?

SUGGESTED EXERCISES

1. Add:

a. 23
42
19
17

b. 172
236
592
135

c. 697
523
367
986

d. 71 yds
59 yds

e. 247 ft
95 ft

f. 129 min
46 min

2. Subtract:

a. 372
276

b. 659
538

c. 976
197

d. 144 in
76 in

e. 428 lbs
72 lbs

f. 128 oz
39 oz

3. Multiply:

a. 651
x 9

b. 536
x 87

c. 326
x 15

d. 182 lbs
x 9

e. 428 tons
x 5

f. 18 ft
x 41

4. Divide:

a. 36) 1512

b. 19) 1653

c. 64) 1472

d. 40) 1200 hrs

e. 23) 161 tons

f. 428) 5992 yds

5. A box contains one dozen snap-fasteners. How many fasteners would you have if you bought three boxes?

6. A hat requires two yards of material. How much material would you need to make five dozen hats?

7. If hair nets are sold four to a pack, how many packs would a beautician buy if 20 hair nets were needed?

8. Mary, a dressmaking student, made four napkins in one hour. How long would it take her to make one napkin?

Lesson 2

Aim

To introduce the Customary System of measurement

Motivation

In football, baseball, track, and other sports, it is important to measure certain distances. What are some of the units used in these sports?

Development

Teacher Activity	Student Activity
1. In discussing responses to the motivation, have the students realize that the units selected for use should correspond to the particular need. Distances in baseball and football, for example, would not be measured in miles.	1. In football, a first-down is a gain of ten yards; in baseball, the distance between the bases and the distance a ball is hit is measured in feet; in track, many of the distances are in parts of a mile.
2. Distribute the information sheet on the Customary Units of Measure. (Note: The units of area, volume, and liquid measure will be studied in detail in Unit V.)	
3. Which units would be used to measure the duration of: a) a trip to the store? b) an adult's life? c) a baby's life? d) the time since 1492?	3. a) minutes or hours b) years c) days, weeks, or months d) years or centuries
4. Which units would be used to measure the length of: a) an airplane trip? b) a building? c) a book?	4. a) miles b) feet or yards c) inches
5. Which units would be used to weigh: a) an ocean liner? b) a person? c) a pack of gum?	5. a) tons b) pounds c) ounces

6. Pose the questions:

a) How many minutes are in four hours?

b) How many feet are there in 16 yards?

c) What operation do we use when converting from a larger unit to a smaller unit?

6.

a) 1 hour = 60 minutes
4 hours = ? minutes

Four hours has 4 x 60, or 240 minutes.

b) 1 yard = 3 feet
16 yards = ? feet

Sixteen yards has 16 x 3, or 48 feet.

c) when converting from a larger unit to a smaller unit, we multiply.

7. Pose the questions:

a) How many feet are there in 60 inches?

b) How many days are there in 72 hours?

c) What operation do we use when converting from a smaller unit to a larger unit?

7.

a) 1 foot = 12 inches
? feet = 60 inches

Sixty is 60/12 or 5 times as great as 12.
Sixty inches equal five feet.

b) 1 day = 24 hours
? days = 72 hours

Seventy-two is 72/24 or 3 times as great as 24.
Seventy-two hours are equal to three days.

c) When converting from a smaller unit to a larger unit, we divide.

Summary

1. How many feet are there in a yard? in a mile?
2. How many days are in a week? this month? a year?
3. How many ounces make a pound?
4. How would you find the number of inches in ten yards?
5. How would you find the number of hours in 300 minutes?

CUSTOMARY UNITS OF MEASUREMENT

LENGTH

1 foot = 12 inches
1 yard = 3 feet
1 yard = 36 inches
1 mile = 5,280 feet

TIME

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days
1 year = 12 months
1 year = 52 weeks (approx.)
1 year = 365 days
1 leap year = 366 days
1 decade = 10 years
1 century = 100 years

ANGLES

1 rotation or revolution = 360 degrees
1 degree = 60 minutes
1 minute = 60 seconds

NAUTICAL

1 fathom = 6 feet
1 mile = 6076 feet

WEIGHT

1 pound = 16 ounces
1 short ton = 2000 pounds

LIQUIDS

1 pint = 16 fluid ounces
1 quart = 2 pints
1 gallon = 4 quarts

AREA

1 sq ft = 144 sq in.
1 sq yd = 9 sq ft
1 acre = 4840 sq yd
1 sq mi = 640 acres

VOLUME

1 cu ft = 1728 cu in
1 cu yd = 27 cu ft

DRY

1 quart = 2 pints
1 peck = 8 quarts
1 bushel = 4 pecks

SUGGESTED EXERCISES

1. How many inches are there in:

   a. 3 yards b. 9 yards c. 1 foot

   d. 7 feet e. 12 feet f. 100 feet

2. How many yards are there in:

   a. 36 inches b. 18 feet c. 72 inches

   d. 1 mile e. 144 inches f. 24 feet

3. How many ounces are there in:

   a. 2 pounds b. 8 pounds c. 1 ton

4. How many hours are there in:

   a. 3 days b. 1 week c. 3 weeks

   d. 120 minutes e. 600 minutes f. 3600 seconds

   g. a leap year h. the month of April

5. If a statement is written with the correct units, write the word CORRECT. If a statement is written with incorrect units, cross out the error and substitute the correct unit.

   a. A child weighs 70 minutes._____

   b. The bag of pears weighed 7 pounds._____

   c. The gasoline tank held 8 yards._____

   d. Sam held his breath underwater for 80 years._____

   e. Judy's new LP record contains 49 ounces of music._____

   f. The car weighed 2500 degrees._____

   g. Gerry had a temperature of 102 centuries._____

   h. The postage stamp had a length of one inch._____

   i. Slim weighs 140 miles._____

   j. The United States is over 2 centuries old._____

6. Read the following paragraph:

A teenager took her brother Mike to the zoo. Mike was only three feet tall, but he acted as if he were a grown person. He showed no fear in approaching the lion's cage, where a small 15-pound lion cub was snoring comfortably away. Mike showed similar bravery in feeding the elephants and making faces at the monkeys. He would have continued the tour, but the 90-degree heat was more than he could bear. The trip home took almost two hours and, thoroughly fatigued, the little "hunter" fell asleep just as he had finished drinking a pint of milk to regain his strength for a future "safari."

a. Name 5 units of measure found in the paragraph.

b. Name 5 instruments that measure the units you listed.

Lesson 3

Aim

To introduce the metric system of measurement

Motivation

The Customary Units of Measure (inch, pound, gallon, etc.) are gradually being replaced by metric units. Can you name some metric units and tell where they are being used now?

Development

Teacher Activity	Student Activity
1. Have the students discuss the metric units with which they are already familiar.	1. Possible responses: millimeters (cameras, film) meters (Olympic races) liters (soda bottles) milliliters (medicine) grams (bread, cake packages) milligrams (Vitamin C)
2. Pose the questions: Why has almost every country in the world adopted the metric system? Why are we changing to it?	2. Possible responses: The metric system a) is based upon powers of ten and is easier to use. b) uses the same prefixes for the units of length, weight and capacity. c) uses decimals instead of fractions. d) has units of length, weight, and capacity which are related to each other. e) is a needed set of standards to be used on an international basis (SI).
3. Distribute the information sheet on metric prefixes.	

PREFIXES OF METRIC UNITS

The following prefixes, when combined with the basic unit names, give the multiples and submultiples used in the metric system.

The unit name meter, with the prefix kilo, becomes kilometer, meaning 1,000 meters (1 km = 1,000 m). When combined with the prefix milli, it becomes millimeter, meaning one thousandth of a meter (1 mm = 0.001 m).

The unit name gram, with the prefix kilo, becomes kilogram, meaning 1,000 grams. (1 kg = 1,000 g). When combined with the prefix milli, it becomes milligram, meaning one thousandth of a gram (1 mg = 0.001 g).

The unit name liter, with the prefix kilo, becomes kiloliter, meaning 1,000 liters (1 kl = 1,000 l). When combined with the prefix milli, it becomes milliliter, meaning one thousandth of a liter (1 ml = 0.001 l).

Prefix	Multiplication Factor	Symbol
mega	1,000,000 (10^6) (one million)	M
kilo	1,000 (10^3) (one thousand)	k
hecto	100 (10^2) (one hundred)	h
deka	10 (10^1) (ten)	da
deci	0.1 (10^{-1}) (one tenth)	d
centi	0.01 (10^{-2}) (one hundredth)	c
milli	0.001 (10^{-3}) (one thousandth)	m
micro	0.000001 (10^{-6}) (one millionth)	μ

SUGGESTED EXERCISES

1. Complete each statent in Column A by choosing the appropriate unit from Column B.

Column A	Column B
a) The plane averaged over 700 ____ per hour.	a) cm
b) Alan's new car weighs about 2200 ____.	b) m
c) Lou ate a dozen donuts and drank nearly one ____ of milk.	c) km
d) Sal drew two pictures, five ____ apart, in his notebook.	d) mg
e) The nurse put three ____ of medicine into a glass of water.	e) g
f) On his diet, Tony is allowed only 50 ___ of salt per week.	f) kg
g) Last winter our school used about 35 ____ of oil for heating.	g) ml
h) My pencil weighs only 12 ____.	h) l
i) Chet won the 100 ____ dash.	i) kl

2. How many centimeters are there in:

   a. 1 meter b. 2 meters c. 1 kilometer

3. How many millimeters are there in:

   a. 10 centimeters b. 1 meter c. 150 centimeters

4. How many milligrams are there in:

   a. 1 centigram b. 1 gram c. 1 kilogram

5. A nurse administered 44 doses of a cough syrup. How many grams were used if one dose equals four grams?

6. A beauty shop operator uses 15 tubes of hair cream a day, each containing 10 grams. How many grams of hair cream will be used in five days?

Lesson 4

Aim

To perform the fundamental operations with units of measure

Motivation

Fred's Furniture Fair purchased 20 minutes of advertising on a local radio station. On Monday, 5 minutes 40 seconds of commercials were broadcast. On Tuesday, 7 minutes 30 seconds were broadcast. How much more time is Fred entitled to?

Development

Teacher Activity	Student Activity *
1. a) Elicit the need to first find the total time used. Have the students realize that exchange with units of measure requires a knowledge and use of the relationships shown in a table of measures.	1. a) Add: 5 min 40 sec 7 min 30 sec 12 min 70 sec = 12 min + 60 sec + 10 sec = 12 min + 1 min + 10 sec = 13 min 10 sec (time used)
b) After rereading the problem, elicit that the time still remaining can now be found by using subtraction. Develop the understanding that 20 minutes is equivalent to 19 minutes 60 seconds, and that regrouping may be necessary.	b) Subtract: 20 min 13 min 10 sec becomes: Subtract: 19 min 60 sec 13 min 10 sec 6 min 50 sec Fred is entitled to another 6 minutes 50 seconds.
2. Pose the problem: Subtract 12 ft 7 in. from 31 ft 4 in.	2. 31 ft 4 in. = 30 ft + 1 ft + 4 in. = 30 ft + 12 in. + 4 in. = 30 ft + 16 in. Therefore, 31 ft 4 in. = 30 ft 16 in. 12 ft 7 in. = 12 ft 7 in. 18 ft 9 in.

* Operations may be done by regrouping units or by converting all items in a problem into the smallest unit. The units expressed in the answer, therefore, may vary.

3. Pose the problem: What is the weight of six books, each of which weighs 2 lbs 8 oz?

3.
$$\begin{array}{r} 2\text{ lb }\ 8\text{ oz} \\ \times\ \ 6 \\ \hline 12\text{ lb }48\text{ oz} \end{array}$$

Since 48 oz = 3 lb,

12 lb 48 oz = 15 lb.

The books weigh 15 lb.

4. Develop division: A board 17 ft 6 in. long is to be cut into five equal parts. How long will each piece be? (Disregard waste.)

4.
$$\begin{array}{r} 3\text{ ft }\ 6\text{ in.} \\ 5\ \overline{)\ 17\text{ ft }\ 6\text{ in.}} \\ \underline{15\text{ ft}} \\ 2\text{ ft }\ 6\text{ in.} = 30\text{ in} \\ \underline{30\text{ in}} \\ 0 \end{array}$$

5. Have the students extend their understandings to include computation with metric units.

a) Add:
$$\begin{array}{r} 4\text{ km }800\text{ m} \\ \underline{2\text{ km }500\text{ m}} \end{array}$$

b) Subtract:
$$\begin{array}{r} 6\text{ m }\ 8\text{ cm} \\ \underline{1\text{ m }30\text{ cm}} \end{array}$$

c) Multiply:
$$\begin{array}{r} 3\text{ kg }250\text{ g} \\ \times\ \ 6 \end{array}$$

d) Divide:
$$4\ \overline{)\ 3\text{ m }22\text{ cm}}$$

5. a) Add:
$$\begin{array}{r} 4\text{ km }800\text{ m} \\ \underline{2\text{ km }500\text{ m}} \\ 6\text{ km }1300\text{ m} = 7\text{ km }300\text{ m} \end{array}$$

b) Because 1 m = 100 cm, and 6 m = 600 cm, we can write:
$$\begin{array}{r} 6\text{ m }\ 8\text{ cm} = \ 608\text{ cm} \\ \underline{1\text{ m }30\text{ cm} = \ 130\text{ cm}} \\ 478\text{ cm} \end{array}$$
or 4 m 78 cm.

c)
$$\begin{array}{r} 3\text{ kg }250\text{ g} \\ \times\ \ 6 \\ \hline 18\text{ kg }1500\text{ g} = 19\text{ kg }500\text{ g} \end{array}$$

d) 3 m 22 cm = 322 cm = 3220 mm
$$4\ \overline{)\ 3220\text{ mm}}\quad 805\text{ mm}$$

805 mm = 80 cm 5 mm

Summary

1. How many feet long is an object measuring 5 ft 12 in.?
2. How many ounces are there in one pound?
3. How many millimeters are there in one meter?
4. How many centigrams of Vitamin E are needed to make one gram?

SUGGESTED EXERCISES

1. Add. Express each sum in simplest form (units may vary).

a. 6 ft 5 in
+ 3 ft 4 in

b. 9 ft 9 in
+ 2 ft 8 in

c. 3 yd 2 ft
+ 8 yd 2 ft

d. 3 kg 40 g
+ 1 kg 60 g

e. 5 cm 9 mm
+ 2 cm 6 mm

f. 3 kl 800 l
+ 7 kl 600 l

2. Subtract. Express each difference in simplest form.

a. 9 lb 14 oz
− 4 lb 5 oz

b. 8 min 10 sec
− 4 min 40 sec

c. 12 ft 6 in
− 3 ft 9 in

d. 3 l 25 ml
− 1 l 25 ml

e. 4 km 200 m
− 1 km 900 m

f. 9 m 8 cm
− 2 m 38 cm

3. Multiply. Express each product in simplest form.

a. 6 ft 9 in
x 8

b. 6 hr 35 min
x 6

c. 8 lb 10 oz
x 10

d. 4 km 600 m
x 5

e. 8 kg 200 g
x 6

f. 25 L
x 16

4. Divide.

a. $3\overline{)9 \text{ lb } 15 \text{ oz}}$

b. $4\overline{)5 \text{ ft } 8 \text{ in}}$

c. $5\overline{)7 \text{ hr } 10 \text{ min}}$

d. $5\overline{)3 \text{ km } 400 \text{ m}}$

e. $6\overline{)8 \text{ kg } 400 \text{ g}}$

f. $16\overline{)8 \text{ l}}$

Lesson 5

Aim

To develop skills in the Customary System of linear measurement

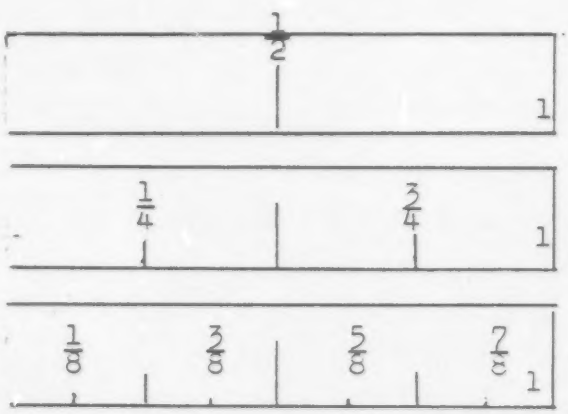
Motivation

Using your ruler, find the length and width of a page from your loose-leaf notebook (or other object distributed for measurement).

Materials

Twelve-inch rulers, yardstick, wooden blocks or other objects to be measured

Development

Teacher Activity	Student Activity
1. Use the results from the motivation problem to evaluate the pupils' ability to use the ruler.	
2. a) What are the most common units of measure used to determine length in the Customary System?	2. a) The inch, the foot, the yard, and the mile.
b) Were you able to measure the loose-leaf paper using only these units?	b) Fractions of an inch were required.
c) Develop the fractional units of an inch in sequence, using an overhead projector, the chalkboard, or an enlarged model of an inch.	c) 
d) Pose questions such as: How many fourths of an inch are in a whole inch? in a half inch? How many eighths of an inch are in one inch? in a half inch? a quarter inch?	

3. When each eighth of an inch is subdivided into two equal parts, what unit do we obtain?

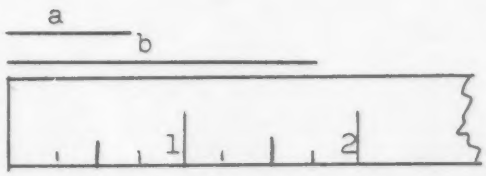
   Extend the development to include sixteenths of an inch.

3. Sixteenths of an inch.

4. Using the ruler, elicit that the same sequence of fractional units appears after each inch.

5. Have the students identify lengths from the model of the inch.

   What is the length of segment a? b?

5. 

6. Have the students measure other objects, drawings, or line segments.

Summary

1. Into how many equal parts is each inch on our ruler subdivided?

2. Which mark lies exactly midway between the 3 inch mark and the 4 inch mark on a ruler?

3. Which mark lies exactly midway between the 6 1/2 inch mark and the 7 inch mark?

SUGGESTED EXERCISES

1. In a brief paragraph, answer the following question.

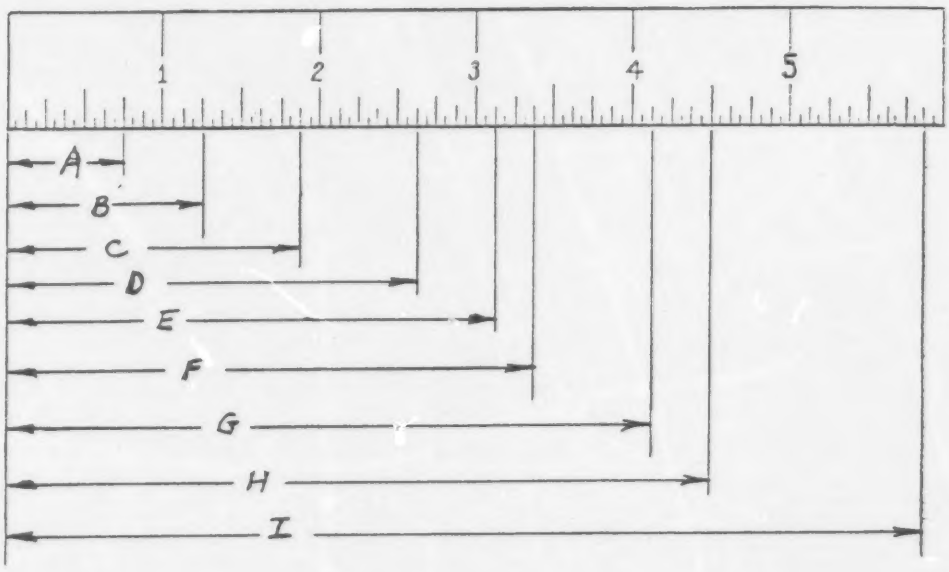
   If the yard is the standard measure of length in our Customary System, why do we need other units such as the inch and the mile?

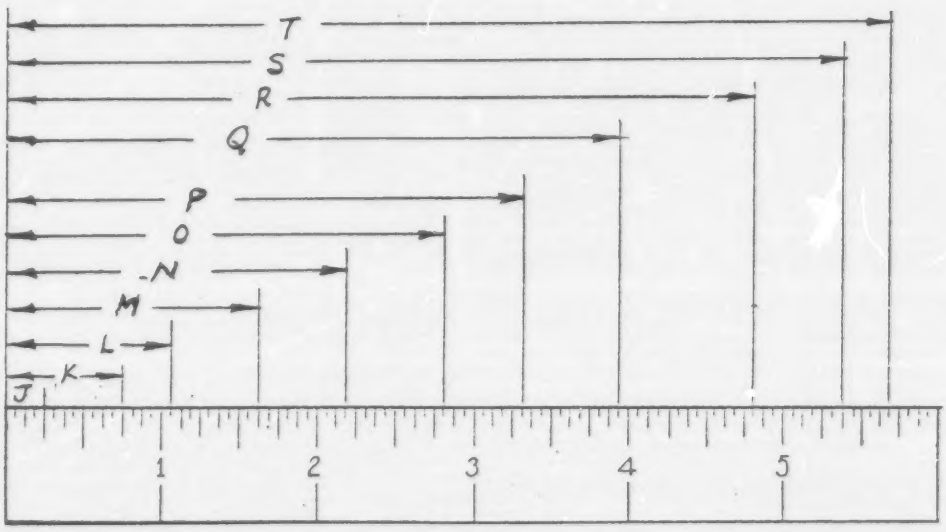
2. An airplane can dive one mile in two seconds. At that rate, how far can it dive in eight seconds?

3. Marty ran 12,320 yards in an hour. How many miles per hour did he run?

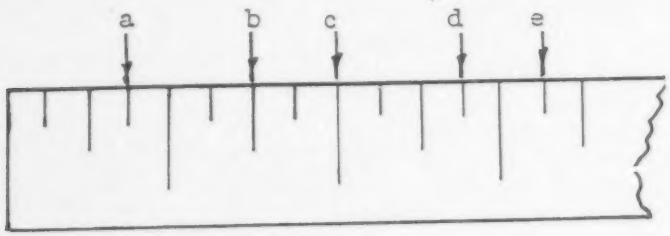
SUGGESTED EXERCISES

4. Give the scale reading for each of the lettered dimensions.



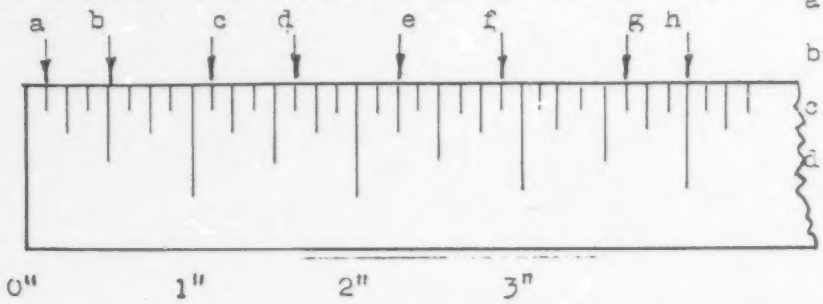


5. On the ruler shown below, read the lengths indicated by arrows at points a, b, c, d, and e.



a = _____
b = _____
c = _____
d = _____
e = _____

6. On the ruler shown below, read the lengths indicated at each arrow:



a = _____ e = _____
b = _____ f = _____
c = _____ g = _____
d = _____ h = _____

Lesson 6

Aim

To develop skills in the use of the metric ruler

Motivation

The metric system of measurement is based upon a unit of measure called a meter. A meter is just a little longer than one of the units in the Customary System. Can you name that unit?

Materials

Meter stick, metric rulers, objects to measure (blocks, line segments, cardboard figures, etc.)

Development

Teacher Activity	Student Activity
1. Discuss the motivation. Use a meter stick to help students visualize and recognize the basic unit of metric length - the meter.	1. The meter is a little longer than a yard.
2. Show the students that the meter is subdivided into 100 equal, numbered units. Have the students relate this unit - the centimeter - to the numbered units on their metric rulers.	2. 1 meter = 100 centimeters
3. Elicit that each centimeter is subdivided into ten equal parts - each of which is a millimeter.	3. 1 centimeter = 10 millimeters 1 meter = 1000 millimeters
4. How long is your metric ruler?	4. Twelve inch rulers have a length of 30 cm 4 mm, or 304 mm.
5. Distribute objects for measurement.	

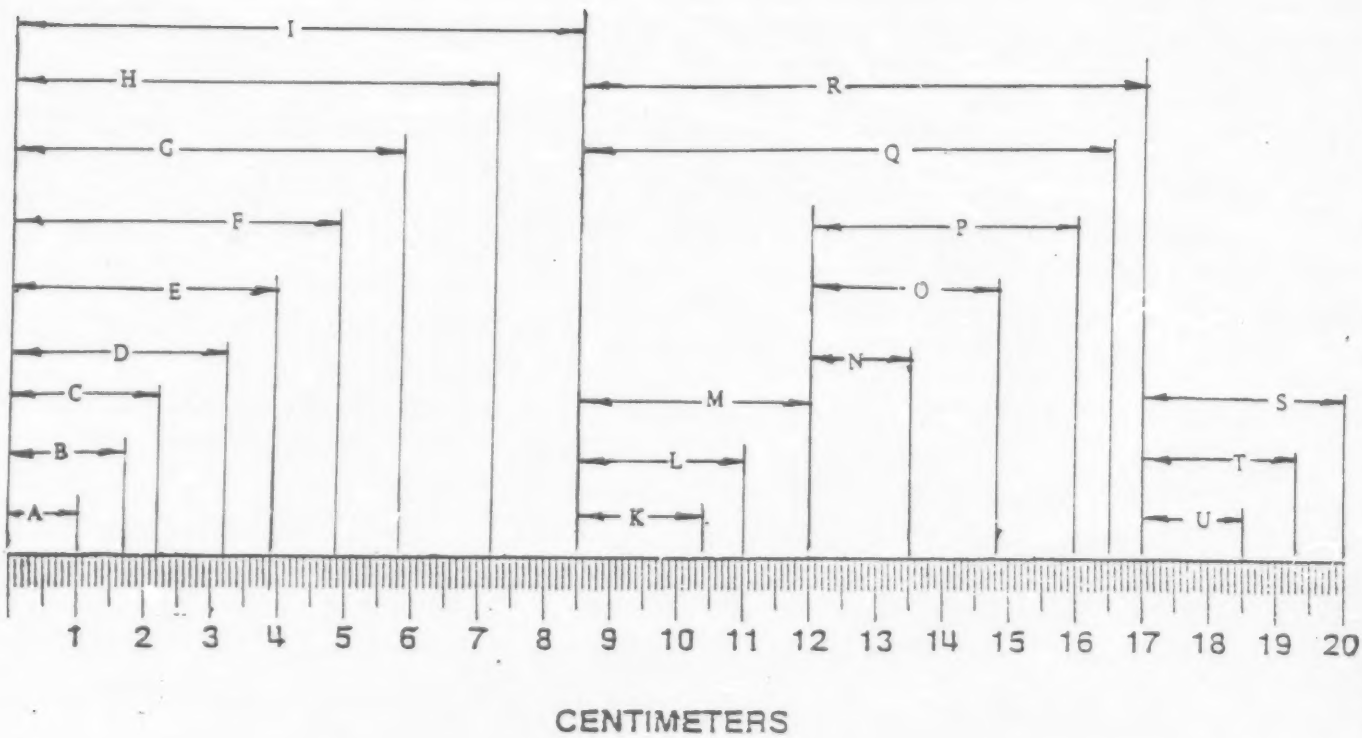
6. Have the students use a meter stick to measure larger objects in the room.

Summary

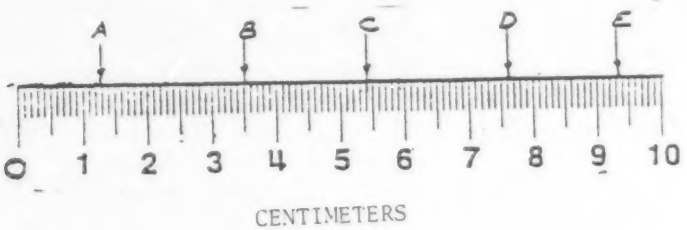
1. How many centimeters are there in one meter?
2. How many millimeters are there in one meter?
3. How many millimeters are equal to one centimeter?
4. Change 21 centimeters to millimeters.

SUGGESTED EXERCISES

1. Give the length of each of the lettered line segments.



2. On the ruler pictured, read the length from the zero point to the point indicated by each arrow.



A _____
B _____
C _____
D _____
E _____

3. Complete each statement:

a. 27 cm = _____ mm.

b. 84 cm = _____ mm.

c. 5 cm 6 mm = _____ mm.

d. 14 cm 8 mm = _____ mm.

e. 135 cm = _____ mm.

f. 3490 cm = _____ mm.

g. 450 mm = _____ cm.

h. 130 mm = _____ cm.

i. 425 mm = _____ cm.

j. 3m = _____ cm.

4. Using a metric ruler, measure the length of each segment.

a. _____

b. _____

c. _____

d. _____

Lesson 7

Aim

To read the micrometer

Motivation

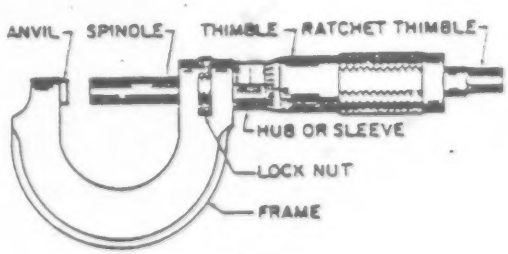
Using the standard machinist scale (ruler), determine if there is a difference in tne diameters of these two rods. (Have two cold rolled steel rods with diameters of .320" and .339".)

Materials

Demonstration micrometer, regular micrometers, steel rods with diameters of .320" and .339"

Development

Teacher Activity	Student Activity
1. Point out to students that, using the standard scale, there does not appear to be a difference in the diameters of the two rods. Inform the students that in many career areas, it is necessary to make measurements with a high degree of precision. For example, moving parts in gears, automobile cylinders, and spark plugs will not function properly if the size of the parts varies more than a few thousandths of an inch. One instrument which can be used to measure accurately to 1/1000 of an inch is the micrometer.	



Micrometer

2. Use a demonstration micrometer, individual pupil micrometers, overhead projector, or a diagram to discuss and explain the various parts of the micrometer and their use.

3. a) The object to be measured is placed between the anvil and the spindle. As the thimble is rotated, it slides along the hub or sleeve.

b) Each numbered division on the sleeve is equal to .1 inch (or 1/10 inch).

Because the micrometer measures to 1/1000 of an inch (.001 in.), it is desirable to express all our units as thousandths, or three-place decimals.

b) .1 in = .100 in

c) Into how many equal parts is each of these numbered divisions subdivided?

c) Each numbered division, (or each .100 inch) is subdivided into four equal parts.

d) What measurement will each of these subdivisions represent?

d) $\frac{.100}{4}$ = .025 inch

4. a) When the thimble is rotated one full turn, how far does it slide along the sleeve?

4. a) One rotation of the thimble slides it .025 inches.

b) How many divisions are marked on the scale on the thimble?

b) The thimble has 25 divisions.

c) If one rotation is equal to .025 inch, and there are 25 divisions, what does each division represent.

c) $\frac{.025}{25}$ = .001 inch

5. a) If we measure an object, and the mark labelled "4" is just visible on the hub, what is the size of the object?

5. a) .400 inch

b) If we now open the micrometer one full turn, to the first subdivision after the 4 , what is the size indicated?

b) $\begin{array}{r} .400 \\ + \ .025 \\ \hline .425 \text{ inch} \end{array}$

c) If the line on the thimble that is closest to the center line on the hub is "15", what would that make the total reading?

c) .400
.025
+ .015
.440 inch

6. If the reading on the hub is between .650 and .675, and the nearest line on the thimble is 13, what is the reading?

6. .650
+ .013
.663 inch

Summary

1. What is the reading for each numbered division on the hub?
2. What is the reading for each subdivision on the hub?
3. What is the reading for each division on the thimble?

SUGGESTED EXERCISES (units may vary)

1. Add.

a. 6 ft 4 in
5 ft 2 in
7 ft 9 in

b. 12 lb 9 oz
14 lb 4 oz
3 lb 8 oz

c. 7 hr 25 min
8 hr 40 min
5 hr 55 min

2. Subtract.

a. 9 ft 5 in
3 ft 7 in

b. 15 km 200 m
2 km 800 m

c. 30 min 5 sec
20 min 15 sec

3. Perform the indicated operation.

a. 3 yd 1 ft
x 9

b. 4) 17 ft 8 in

c. 2 kg 300 g
x 5

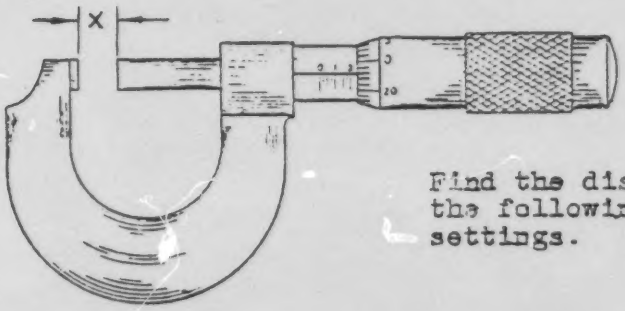
4. Out of a salary of $292.50, Mr. Jones saves $83.20 for rent. How much money does he have left to spend for other items?

5. A student spent $0.05 for a pencil, $1.42 for a notebook, and $9.49 for a briefcase. Find her total expenses.

6. A salesperson received a five-dollar bill for a purchase of $3.95. How much change should he return?

1.

PROBLEMS IN READING A MICROMETER



Find the distance "X" for the following micrometer settings.

NOTE: Where possible, secure a micrometer, set at the indicated positions, and note the reading in each instance.

	Reading on sleeve is between	Nearest line on thimble	Reading
a.	.000 and .025	6	
b.	.075 and .100	14	
c.	.125 and .150	24	
d.	.250 and .275	5	
e.	.325 and .350	15	
f.	.450 and .475	23	
g.	.400 and .425	4	
h.	.575 and .600	16	
i.	.525 and .550	22	
j.	.675 and .700	3	
k.	.600 and .625	17	
l.	.700 and .725	21	
m.	.775 and .800	2	
n.	.825 and .850	18	
o.	.900 and .925	20	
p.	.975 and 1.000	1	
q.	.800 and [illegible]	19	
r.	.800 and .825	7	
s.	.750 and .775	13	
t.	.625 and .650	8	

UNIT II

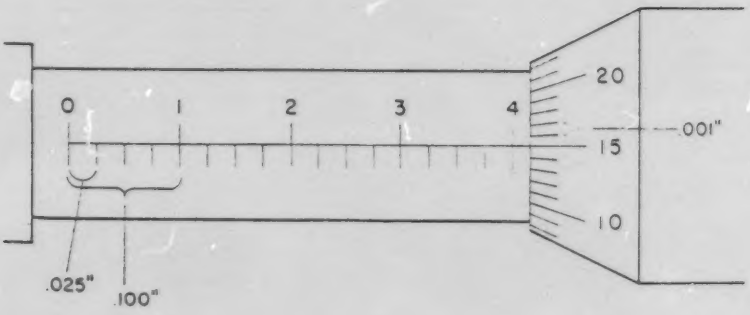
Lesson 8

Aim

To develop skill in the use of the micrometer

Motivation

Read the micrometer setting in the diagram below:



Materials

Demonstration micrometer and regular micrometers.

Development

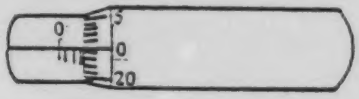
Teacher Activity	Student Activity
1. To read the micrometer elicit from the students the three observations they must make.	1. To read the setting of a micrometer, you must observe three things: the greatest numeral on the hub the number of additional spaces visible on the hub the number of spaces indicated on the thimble
2. a) What is the greatest numeral visible on the hub?	2.a) 4
b) What additional whole spaces are fully visible on the hub?	b) No additional whole spaces are visible
c) What spaces are indicated on the thimble?	c) 15

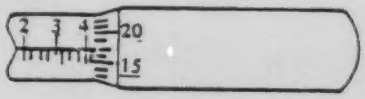
3.	Using the above information, guide the students to convert the readings to decimal parts of an inch.	3.	4 x .100" = .400" 0 x .025" = .000" 15 x .001" = .015" Total: .415 inch
4.	Pose the problem: If three full spaces beyond the seven are visible on the hub, and a 23 is indicated on the thimble, what is the measurement?	4.	7 x .100" = .700" 3 x .025" = .075" 23 x .001" = .023" Total: .798 inch
5.	Distribute micrometers to small groups of two or three students. Distribute bolts or other appropriate objects for measurement. Demonstrate the careful use of the micrometer as a precision tool.		

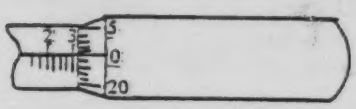
Summary

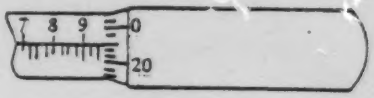
1. Why must a micrometer be handled carefully?
2. What will happen if too much pressure is applied to the thimble?
3. What measurement does each numeral on the hub represent?
4. What measurement does each subdivision on the hub represent?
5. What measurement does each division on the thimble represent?

5. What measurement is represented?

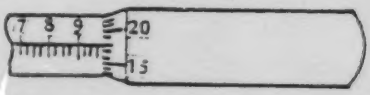
a  a _____

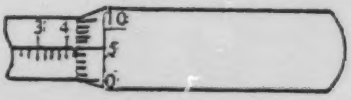
b  b _____

c  c _____

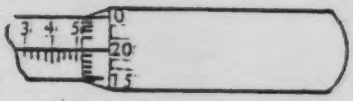
d  d _____

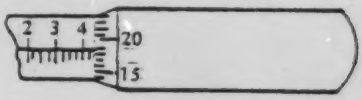
e e _____

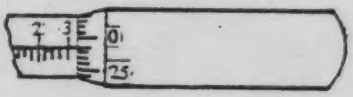
f  f _____

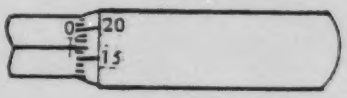
g  g _____

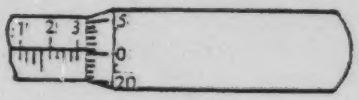
h h _____

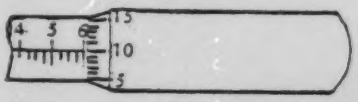
i  i _____

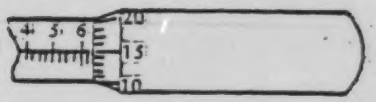
j  j _____

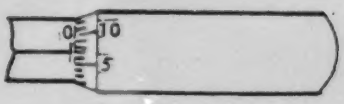
k  k _____

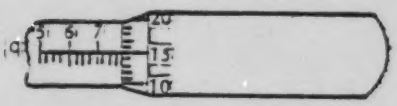
l  l _____

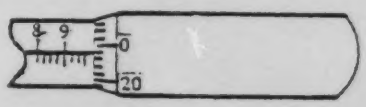
m  m _____

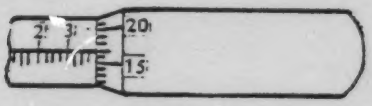
n  n _____

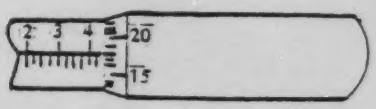
o  o _____

p  p _____

q  q _____

r  r _____

s  s _____

t  t _____

Lesson 9

Aim

To determine the upper and lower limits of a tolerance interval

Motivation

Which will give the measurement closest to the actual weight?
a) weighing a car to the nearest ton
b) weighing a car to the nearest pound
c) weighing a car to the nearest ounce

Development

Teacher Activity	Student Activity
1. a) Discuss the motivation problem.	1. a) Weighing a car to the nearest ounce would give a measurement closest to the actual weight.
b) Have the students consider the possibility of using a scale that can weigh to the nearest 0.1 ounce, or 0.01 ounce. Would the measurement be even closer?	b) The smaller the unit of measure, the closer the reading will be to the actual weight. The smaller the unit of measure, the more precise the measurement.
c) Have the students discuss the possibility of always being able to construct a measuring instrument with smaller and smaller units. In each case, the measurement would be to the nearest of the units used. For this reason we say all measurements are approximate.	c) No measurement is ever exact. Measurements are always approximate.
2. a) Since measurement is always approximate, it is extremely important in industry to know the amount by which the measure of an object may vary from a given size and still work properly.	
b) The amount a measurement may vary from the desired size is called its tolerance.	

3. For a 50 yard race, the desired length of track was 150 ft ± 1 in. (± = + or -)

   a) What is the upper limit?

   b) What is the lower limit?

   c) What are the acceptable limits?

   d) Inform the students that the upper and lower limits give us the <u>tolerance interval</u>.

3.

a) 150 ft + 1 in , or 150' 1"

b) 150 ft - 1 in , or 149' 11"

c) From 149' 11" to 150 ft 1"

d) From 149' 11" to 150 ft 1"

4. What are the upper and lower limits for a blueprint measurement stated as 2.375" ± 0.050"

4.

$$\begin{array}{r} 2.375" \\ +\ 0.050" \\ \hline 2.425" \end{array}$$ upper limit

$$\begin{array}{r} 2.375" \\ -\ 0.050" \\ \hline 2.325" \end{array}$$ lower limit

5. The desired size of a part for a jet engine is given as 1.150" ± 0.005". Is a part measuring 1.144" acceptable, i.e., within the tolerance interval?

5.

$$\begin{array}{r} 1.150" \\ -\ 0.005" \\ \hline 1.145" \end{array}$$ lower limit

1.144" is not acceptable

6. Sometimes the tolerance is allowed in one direction only. What are the upper and lower limits for a measurement stated as 4.25" + 0.02" ?

6.

$$\begin{array}{r} 4.25" \\ +\ 0.02" \\ \hline 4.27" \end{array}$$ upper limit

4.25" lower limit

<u>Summary</u>

1. Is measurement approximate or exact?
2. How can we obtain a more precise measurement?
3. What do we mean by the tolerance for a measurement?
4. The desired size for a pretzel stick is 10 in. ± 1 in. What is the tolerance interval for a pretzel stick?

SUGGESTED EXERCISES

1. Tell whether the number in each statement is exact or approximate.

   a. He bought two bottles of soda.

   b. He drank two liters of soda.

   c. Mary weighs 110½ pounds.

   d. There are 36 inches in one yard.

   e. The rainfall measured 2 7/8 inches.

   f. My car averages 22.3 miles per gallon.

   g. Fred's car is 28 feet long.

   h. Fred's class has 28 students.

   i. I woke up at exactly 7:00 a.m.

   j. The temperature was 31°.

   k. The nurse weighed nine babies.

   l. The baby weighed nine pounds.

2. Determine the upper limit and the lower limit for each:

		Upper limit	Lower limit
a.	1.045" ± 0.003"	_____	_____
b.	.978" ± .001"	_____	_____
c.	.625" ± .005"	_____	_____
d.	2.375" + 0.005"	_____	_____
e.	.881" - .002"	_____	_____
f.	2.250" + 0.003" - 0.001"	_____	_____
g.	.162" + .004" - .002"	_____	_____

UNIT II

Lesson 10

Aim

To measure temperature in the Fahrenheit and Celsius scales.

Motivation

Suppose that you are on vacation in Mexico City. You do not feel well and decide to see a doctor in the hotel. After examining you, he tells you that your temperature is 40 degrees. What does this temperature reading indicate? Do you have a fever?

Materials

Two thermometers, one in Fahrenheit scale and the other in Celsius scale, ice, burner, water, and glass beaker

Development

Teacher Activity

1. Discussion of the motivation should lead to the two basic scales used in measurement.

2.a. Draw water from the tap and take temperature readings in both the Fahrenheit and Celsius scales.
2.b-d. Add ice to the beaker and have students take three readings while adding the ice.
2.e-g. Allow water to reach maximum coolness. Record this temperature as the freezing point.
2.h-j. Discard water and refill with tap water. Heat gradually and take three additional readings as the water is heated. Record the highest temperature as the boiling point.

Note that cold or coolness should be regarded in terms of the absence or lowering of heat.

Student Activity

	Celsius Readings	Fahr. Readings
a		
b		
c		
d		
e		
f		
g		
h		
i		
j		

3. a) Have the students determine at which temperature reading in Celsius and Fahrenheit water freezes.

b) Have the students determine at which temperature reading in Celsius and Fahrenheit water boils.

4. Inform students that the relationship between Celsius and Fahrenheit readings is given by the conversion formulas:

$F = \frac{9}{5} C + 32 \qquad C = \frac{5}{9} (F - 32)$

Using the appropriate formula, have the students convert the 40° C reading in the motivation to Fahrenheit degrees.

3. a) The freezing point of water is 0° C and 32° F.

b) The boiling point of water is 100° C and 212° F.

4. $C = 40^\circ$

$F = \frac{9}{5} C + 32$

$F = \frac{9}{5} (40) + 32$

$F = 72 + 32$

$F = 104^\circ$

Summary

1. What is the freezing point of water on the Celsius scale? On the Fahrenheit scale?

2. What is the boiling point of water on the Celsius scale? On the Fahrenheit scale?

SUGGESTED EXERCISES

1. Use the formula $F = \frac{9}{5} C + 32$ to change the following Celsius readings to their Fahrenheit equivalents:

a. 20° C b. 5° C c. 35° C d. 50° C

2. Use the formula $C = \frac{5}{9} (F - 32)$ to change the following Fahrenheit readings to their Celsius equivalents:

a. 86° F b. 23° F c. 68° F d. 5° F

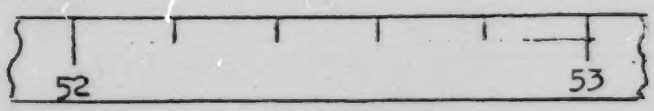
3. The two temperature scales in common use are the _____ and the _____ thermometers.

4. Using either the word Fahrenheit or Celsius, complete each of the following sentences.

   a. Water boils at 100°_____.
   b. On a hot day, New York City may reach a temperature of 95°_____
   c. The nurse told a doctor that her patient had a temperature of 102.2°_____.
   d. An air conditioner may keep the room temperature at a comfortable 66°_____.
   e. It is easy to remember the temperature at which water freezes to ice. This temperature happens to be 0°_____.

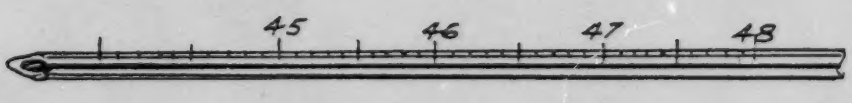
5. On the thermometer shown below, draw arrowheads at each of the following temperatures.

   a) 52° b) 52.2° c) 52.8° d) 52.3° e) 52.7°

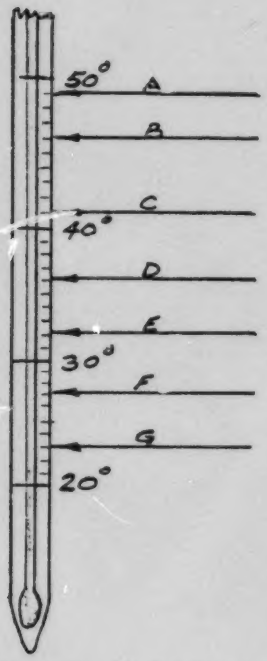


6. On the thermometer shown below, draw arrowheads at each of the following temperatures.

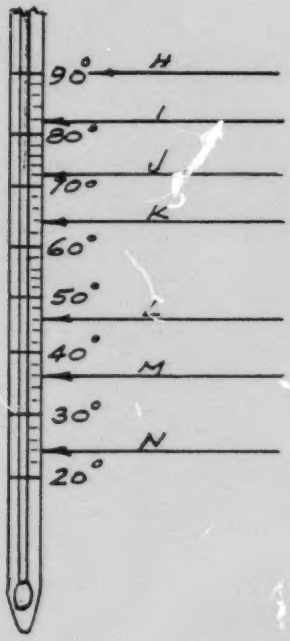
   a) 45 b) 45.3 c) 45.8 d) 46.5 e) 46.7 F) 46.9



7. Read the Celsius and Fahrenheit thermometers at each of the arrowheads and record the values.



DEGREES CELSIUS



DEGREES FAHRENHEIT

A. _____

B. _____

C. _____

D. _____

E. _____

F. _____

G. _____

H. _____

I. _____

J. _____

K. _____

L. _____

M. _____

N. _____

Unit III

FRACTIONS, DECIMALS, AND PERCENTS

Lesson 1	To learn the meaning of fractions (GHSM p. 18)*
Lesson 2	To write equivalent fractions in higher and lower terms (GHSM p. 21)*
Lesson 3	To add and subtract fractions with the same denominators (GHSM p. 19)*
Lesson 4	To add and subtract fractions with different denominators (GHSM pp. 19; 21-24)*
Lesson 5	To add mixed numerals (GHSM pp. 26-27)*
Lesson 6	To subtract mixed numerals (GHSM p. 27)*
Lesson 7	To multiply fractions (GHSM pp. 26-27)*
Lesson 8	To divide fractions and mixed numerals (GHSM pp. 26-27)*
Lesson 9	To convert a fraction to a decimal (GHSM pp. 28-31)*
Lesson 10	To convert a decimal to a fraction (GHSM pp. 28-31)*
Lesson 11	To add and subtract decimals (GHSM pp. 28-31)*
Lesson 12	To multiply decimals (GHSM pp. 28-31)*
Lesson 13	To divide decimal numerals (GHSM pp. 28-31)*
Lesson 14	To relate fractions, decimals, and percents (GHSM pp. 27-30; 44-45)*
Lesson 15	To find a percent of a number (GHSM pp. 44-45)*
Lesson 16	To find what percent one number is of another (GHSM pp. 44-45)*

*General High School Mathematics, New York State Department of Education, August 1978.

UNIT III

FRACTIONS, DECIMALS AND PERCENTS

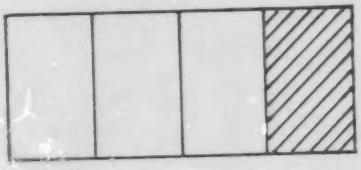
Lesson 1

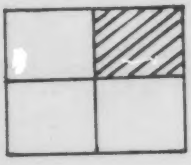
Aim

To learn the meaning of fractions

Motivation

What relationship exists among the shaded areas shown?





Development

Teacher Activity	Student Activity
1. a) Discuss the motivation and elicit the relationship that exists.	1. a) In each diagram, one-fourth of the area has been shaded.
b) How can one-fourth be written in mathematical symbols?	b) One-fourth = $\frac{1}{4}$
2. Discuss the meaning and use of an arithmetic fraction.	2. A fraction is used to represent one or more equal parts into which a whole has been divided.
3. a) What are the two terms of a fraction called? b) In the fraction 1/4, which is the numerator? the denominator?	3. a) The terms of a fraction are its numerator and denominator. b) One is the numerator, four is the denominator.
4. a) What does the denominator of a fraction represent?	4. a) The denominator tells the number of equal parts into which the whole unit has been divided.
b) What does the numerator of a fraction tell us?	b) The numerator tells the number of equal pieces represented.
5. If a unit is divided into eight equal parts, how can we represent: a) one of these parts? b) three of these parts? c) all of these parts? d) one whole unit?	5. a) 1/8 b) 3/8 c) 8/8 d) 8/8 = 1

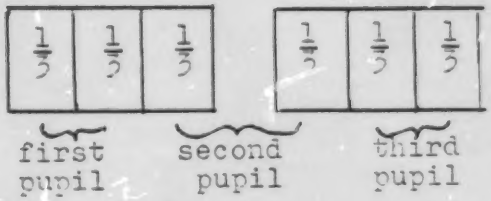
6. a) A fraction may also be used to represent the division of one number by another.

b) If two bars of candy are to be divided equally among three students, how much will each receive?
Elicit that each will receive 2/3 bar of candy.

c) What is 2 divided by 3 ?

6.

b)



c) $2 \div 3 = \frac{2}{3}$

7. Express each as a fraction:

a) 7 divided by 8
b) 6 divided by 6
c) $3 \div 4$
d) $3 \overline{)5}$

7.

a) 7/8
b) 6/6
c) 3/4
d) 5/8

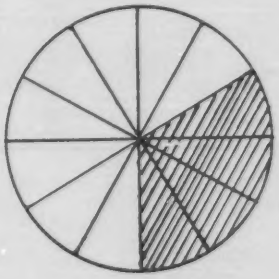
8. Discuss the importance of fractions in the trade areas of particular interest to the students.

Summary

1. In 9/10, what is the numerator? What is the denominator?
2. To show 5/6, into how many equal parts must the whole unit be divided?
3. A pizza was cut into eight equal parts. If you eat five of these parts, what fraction of the pizza have you eaten?
4. How can 'three divided by ten' be expressed as a fraction?

SUGGESTED EXERCISES

1. a. Into how many equal pieces is the pie sliced?
   b. How many pieces are shaded?
   c. Write the shaded part as a fraction of the whole pie.
   d. If only 1 piece were shaded, how would you write that as a fraction?



2. Answer the questions below each of the following diagrams (Figures A-E).

FIGURE A

FIGURE B

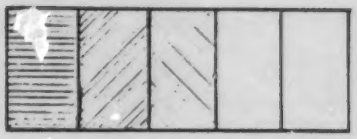
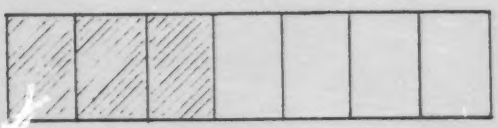


FIGURE C

	A	B	C
a. The figure is divided into _____ equal parts.	a. _____	a. _____	a. _____
b. _____ pieces are shaded.	b. _____	b. _____	b. _____
c. The fractional part shaded is _____	c. _____	c. _____	c. _____
d. If only one part were shaded, the fractional part shaded would be _____.	d. _____	d. _____	d. _____

Figure D



D. a. _____
b. _____
c. _____
d. _____

Figure E

E. a. _____
b. _____
c. _____
e. _____

3. Answer the questions associated with each diagram.

3.

4.

a. Into how many equal parts is the pie divided?

b. How many pieces are shaded?

c. Write the shaded part as a fraction of the whole pie.

d. Write the unshaded part as a fraction of the whole pie.

4. a. Into how many equal parts is the pie divided?

b. How many pieces are shaded?

c. Write the shaded part as a fraction of the whole pie.

d. Write the unshaded part as a fraction of the whole pie.

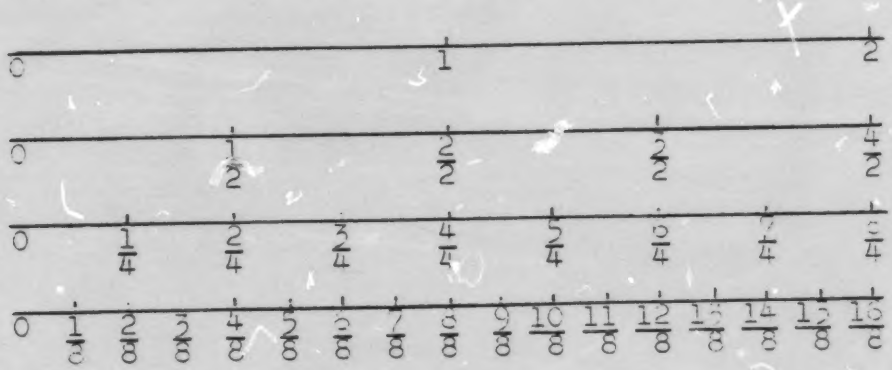
Lesson 2

Aim

To write equivalent fractions in higher and lower terms

Motivation

Use the diagram to find two fractions the same size as 1/2.



Development

Teacher Activity	Student Activity
1. Elicit that fractions with the same size or value are called equivalent fractions.	1. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$
2. a) Which fractions are equivalent to 12/8 ? b) Which fractions are equivalent to one? c) Which fractions are equivalent to 3/4 ?	2. a) $\frac{12}{8} = \frac{6}{4} = \frac{3}{2}$ b) $1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$ c) $\frac{3}{4} = \frac{6}{8}$
3. Recall that the value of a number is not changed if it is multiplied by one. Develop the procedure for obtaining equivalent fractions: $\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$	
4. Have students change 1/2 to an equivalent fraction with a denominator of eight.	4. $\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$
5. Have students generalize the procedure for obtaining an equivalent fraction in higher terms.	5. When the numerator and the denominator are multiplied by the same number we obtain an equivalent fraction.

6. a) Have students consider the equivalent fractions obtained from the motivation.	6. a) $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$
b) Which of these fractions is in simplest form?	b) 1/2 is in simplest form.
7. a) How can we find an equivalent fraction in higher terms?	7. a) By multiplying the numerator and denominator by the same whole number.
b) How can we find an equivalent fraction in lower terms?	b) By dividing the numerator and denominator by the same whole number.
8. a) Develop the procedure for finding an equivalent fraction in simpler form.	8.
b) Have students consider 2/8. What number can be divided exactly into 2 and into 8?	b) Two is a factor of both two and eight.
c) $\frac{2}{8} \div 1 = \frac{2}{8} \div \frac{2}{2} = \frac{1}{4}$	
9. a) Have students list all the numbers which are factors of both 12 and 16.	9. a) The common factors of 12 and 16 are: 1, 2, and 4.
b) Which of these common factors can be used to change 12/16 to an equivalent fraction in simplest form?	b) The greatest common factor, 4, can be used to change 12/16 to simplest form. $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$
10. Have students generalize the procedure for simplifying a fraction to lowest terms.	10. To simplify a fraction, divide the numerator and the denominator by their greatest common factor.

Summary

1. What are equivalent fractions?
2. If the numerator and denominator of a fraction are both multiplied by eight, is the value of the fraction changed? Explain.
3. If the numerator and denominator of a fraction are both divided by a common factor, is the value of the fraction changed? Explain.
4. How can we simplify a fraction to its lowest terms?

SUGGESTED EXERCISES

1. What is the greatest common factor of:

a. 8 and 16	b. 75 and 100	c. 60 and 100
d. 6 and 16	e. 10 and 15	f. 24 and 32
g. 8 and 24	h. 6 and 8	i. 60 and 80

2. Write in simplest form.

a. $\frac{4}{16}$	b. $\frac{6}{16}$	c. $\frac{6}{20}$	d. $\frac{2}{10}$
e. $\frac{4}{24}$	f. $\frac{28}{32}$	g. $\frac{25}{40}$	h. $\frac{3}{24}$
i. $\frac{14}{21}$	j. $\frac{21}{56}$	k. $\frac{14}{16}$	l. $\frac{18}{24}$
m. $\frac{8}{12}$	n. $\frac{9}{12}$	o. $\frac{10}{12}$	p. $\frac{8}{32}$

3. Express each as an equivalent fraction with a denominator of 16.

a. $\frac{7}{8}$	b. $\frac{1}{2}$	c. $\frac{3}{4}$	d. $\frac{5}{8}$
e. $\frac{3}{8}$	f. $\frac{1}{4}$	g. $\frac{3}{8}$	h. $\frac{1}{8}$

4. Draw three columns. Label them "Greater than one whole," "Exactly one," and "Less than one." Then write each of these fractions in the correct column.

a. $\frac{7}{16}$	b. $\frac{8}{8}$	c. $\frac{5}{4}$	d. $\frac{3}{8}$
e. $\frac{1}{2}$	f. $\frac{4}{3}$	g. $\frac{4}{4}$	h. $\frac{16}{8}$
i. $\frac{1}{4}$	j. $\frac{3}{2}$	k. $\frac{2}{2}$	l. $\frac{6}{4}$
m. $\frac{16}{16}$	n. $\frac{11}{8}$	o. $\frac{2}{3}$	p. $\frac{5}{5}$

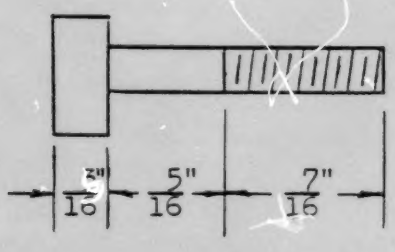
Lesson 3

Aim

To add and subtract fractions with the same denominators

Motivation

Represent the total length of the bolt as a single fraction.



Development

Teacher Activity	Student Activity
1. Have students state the total length of the bolt and explain how they obtained it.	1. Since all the measurements are in sixteenths, we need add only the number of sixteenths. 3 + 5 + 7 = 15 The total length is 15/16 in.
2. Elicit the rule for adding fractions with the same denominator.	2. When adding fractions with the same denominator, add the numerators and retain the common denominator.
3.a) Pose the problem: A rivet 11/16 in. long had 5/16 in. cut from it. What was the length of the remaining piece?	3.a) Five sixteenths from 11 sixteenths leaves six sixteenths. $\frac{11}{16} - \frac{5}{16} = \frac{6}{16}$
b) Recall that we write a fraction in simplest form.	b) $\frac{6}{16} = \frac{3}{8}$ The remaining piece is 3/8 inches long.
4. Elicit the rule for subtracting fractions with the same denominator.	4. When subtracting fractions with the same denominator, subtract the numerators and retain the common denominator.

5. a) Have the students add:
$\frac{7}{16} + \frac{5}{16} + \frac{9}{16}$

b) How many sixteenths are there in a whole unit?

c) How does this sum (21/16) compare with 16/16 ?

d) Elicit that 21/16 is greater than one whole.

e) The sum of a whole number and a fraction is written as a <u>mixed numeral</u>.

5. a) $\frac{7}{16} + \frac{5}{16} + \frac{9}{16} = \frac{21}{16}$

b) $1 = \frac{16}{16}$

c) $\frac{21}{16}$ is greater than $\frac{16}{16}$.

d) $\frac{21}{16} = \frac{16}{16} + \frac{5}{16} = 1 + \frac{5}{16}$

e) $1 + \frac{5}{16} = 1\frac{5}{16}$

6. a) Have students consider these fractions:
$\frac{3}{4}$ $\frac{9}{8}$ $\frac{2}{2}$ $\frac{3}{8}$ $\frac{7}{4}$ $\frac{8}{8}$

b) Which have a value less than one?

c) What do we call fractions with a value less than one?

d) How can we identify a proper fraction?

e) Which of the above fractions are equivalent to one?

f) Which have a value greater than one?

g) What do we call a fraction equal to, or greater than one?

h) How can we identify an improper fraction?

6.

b) $\frac{3}{4}$ and $\frac{3}{8}$

c) Fractions with a value less than one are called <u>proper</u> fractions.

d) In a proper fraction, the numerator is less than the denominator.

e) $\frac{2}{2}$ and $\frac{8}{8}$

f) $\frac{9}{8}$ and $\frac{7}{4}$

g) A fraction equal to, or greater than one, is called an <u>improper</u> fraction.

h) In an improper fraction, the numerator is equal to, or greater than the denominator.

7. Rewrite each improper fraction as an equivalent whole number or mixed numeral.
a) $\frac{8}{8}$ b) $\frac{7}{4}$ c) $\frac{9}{6}$

7. a) $\frac{8}{8} = 1$

b) $\frac{7}{4} = \frac{4}{4} + \frac{3}{4} = 1 + \frac{3}{4} = 1\frac{3}{4}$

$\frac{9}{6} = \frac{6}{6} + \frac{3}{6} = 1 + \frac{3}{6} = 1 + \frac{1}{2} = 1\frac{1}{2}$

Summary

1. How do we add fractions which have the same denominator?
2. How do we subtract fractions which have the same denominator?
3. Is 3/4 a proper or an improper fraction? Explain.
4. Is 15/8 a proper or an improper fraction? Explain.
5. How can we change 5/4 to an equivalent mixed numeral?

Suggested Exercises

1. Write as an equivalent whole number or mixed numeral.

a. $\frac{16}{8}$ b. $\frac{5}{4}$ c. $\frac{3}{2}$ d. $\frac{7}{4}$ e. $\frac{11}{8}$

f. $\frac{4}{4}$ g. $\frac{15}{8}$ h. $\frac{11}{4}$ i. $\frac{9}{2}$ j. $\frac{25}{4}$

2. Add or subtract, as indicated.

a. $\frac{3}{8} + \frac{2}{8}$ b. $\frac{7}{16} + \frac{6}{16}$ c. $\frac{3}{5} + \frac{1}{5}$ d. $\frac{7}{8} - \frac{4}{8}$ e. $\frac{14}{16} - \frac{5}{16}$

3. Add or subtract, as indicated. Simplify your answer when necessary.

a. $\frac{5}{16} + \frac{7}{16}$ b. $\frac{3}{8} + \frac{5}{8}$ c. $\frac{9}{16} + \frac{3}{16}$ d. $\frac{1}{4} + \frac{3}{4}$ e. $\frac{13}{16} + \frac{11}{16}$

f. $\frac{7}{8} - \frac{3}{8}$ g. $\frac{15}{16} - \frac{5}{16}$ h. $\frac{3}{4} - \frac{1}{4}$ i. $\frac{23}{32} - \frac{7}{32}$ j. $\frac{57}{100} - \frac{41}{100}$

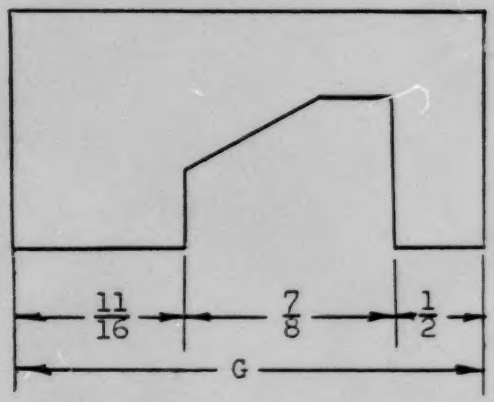
Lesson 4

Aim

To add and subtract fractions that have different denominators

Motivation

Using the template drawing, determine how dimension G can be represented as a single fraction.



Development

Teacher Activity	Student Activity
1. a) Discuss the motivation. What operation is required?	1. a) We must add the three fractions, 7/8, 11/16, and 1/2.
b) Can we add these fractions directly?	b) No, because they have different denominators.
c) Can we change them so they have the same denominator?	c) Yes, by using equivalent fractions.
d) To what common denominator can these three fractions be changed?	d) They can all be changed to sixteenths.
2. a) Recall the procedure for obtaining equivalent fractions.	2. a) We multiply the numerator and the denominator by the same number.
b) $\frac{7}{8} \times 1 = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$	
c) $\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$	

3. Have the students solve the motivation problem.

3. $\frac{7}{8} = \frac{14}{16}$

$\frac{11}{16} = \frac{11}{16}$

$\frac{1}{2} = \frac{8}{16}$

$\frac{33}{16} = \frac{32}{16} + \frac{1}{16} = 2\frac{1}{16}$

4. Since sixteen is the least number we could have used as a common denominator, it is called the <u>least common denominator</u>, or <u>LCD</u>.

The LCD is the smallest number that is divisible by each of the original denominators.

5. a) Have the students consider the subtraction example:

$$\frac{3}{4} - \frac{2}{3}$$

5. a) The denominators are not the same.

b) What is the LCD for 3/4 and 2/3?
Note that the LCD is not one of the denominators.

b) We can use the multiples to see: 3, 6, 9, <u>12</u>
4, 8, <u>12</u>
The LCD is 12.

c) Have students obtain the equivalent equations and subtract.

c) $\frac{3}{4} = \frac{9}{12}$

$\frac{2}{3} = \frac{8}{12}$

$\frac{1}{12}$

<u>Summary</u>

1. How do we add or subtract fractions which have different denominators?

2. What does LCD mean?

3. What is the LCD for the fractions 1/2 and 2/3 ? Explain why?

SUGGESTED EXERCISES

1. Convert each fraction to an equivalent fraction having the denominator indicated:

   a. $\frac{3}{5} = \frac{\quad}{20}$ b. $\frac{3}{4} = \frac{\quad}{12}$ c. $\frac{3}{4} = \frac{\quad}{20}$

   d. $\frac{7}{8} = \frac{\quad}{32}$ e. $\frac{5}{8} = \frac{\quad}{16}$ f. $\frac{7}{10} = \frac{\quad}{100}$

2. Add the following fractions:

   a. $\frac{3}{16} + \frac{5}{16}$ b. $\frac{2}{3} + \frac{4}{5}$ c. $\frac{3}{4} + \frac{3}{5} + \frac{1}{2}$ d. $\frac{1}{4} + \frac{1}{50}$

3. Subtract the following fractions:

   a. $\frac{7}{8} - \frac{3}{8}$ b. $\frac{31}{32} - \frac{3}{8}$ c. $\frac{18}{15} - \frac{3}{5}$ d. $\frac{9}{10} - \frac{1}{2}$

4. Fred ate 1/3 of a pizza, and Pat ate 1/2 of the same pizza. What fractional part of the pizza did the two students eat together? What fractional part of the pizza was left?

5. A carpenter had three boards with thicknesses of 3/8 in., 1/2 in., and 5/16 in., which he glued together for strength. What was the total thickness of the new board?

6. Sue had to insert a pipe with an outside diameter of 7/8 inch into an opening 11/16 inch. How much larger did she have to make the opening?

Lesson 5

Aim

To add mixed numerals

Motivation

Last month, Carmine and Lois made two long trips on their motorbikes. On one trip they drove a distance of 21 1/4 miles round trip to attend a stock car race. The following week they drove 18 3/4 miles round trip to attend a fashion show. What was the total number of miles that they drove on both trips?

Development

Teacher Activity	Student Activity
1. Recall that 21 1/4 and 18 3/4 are called mixed numerals. Elicit from the students that the whole numbers and the fractions are added separately. When necessary, improper fractions are changed to mixed numerals. Finally, the whole number and the fraction are combined into a final sum.	1. $21\frac{1}{4}$ $18\frac{3}{4}$ $39\frac{4}{4} = 39 + 1 = 40$
2. Have students add: $12\frac{3}{4} + 12\frac{5}{8} + 10\frac{1}{2}$ Recall: a) Change fractions to the same denominator. b) Add fractions and whole numbers. c) Simplify improper fractions. d) Combine the whole number and mixed numeral results.	2. $12\frac{3}{4} = 12\frac{6}{8}$ $12\frac{5}{8} = 12\frac{5}{8}$ $10\frac{1}{2} = 10\frac{4}{8}$ $34\frac{15}{8}$ $\frac{15}{8} = 1\frac{7}{8}$ $34 + 1\frac{7}{8} = 35\frac{7}{8}$

Summary

1. In order to add 5 3/4 and 2 9/16, what must we do first?
2. When an answer contains an improper fraction, how can we change it to a mixed numeral?

SUGGESTED EXERCISES

1. Add. Express each sum in simplest form.

a.	b.	c.	d.
$7\frac{3}{4}$	$12\frac{1}{2}$	$7\frac{2}{3}$	$12\frac{9}{16}$
$6\frac{5}{8}$	$7\frac{4}{5}$	$8\frac{1}{4}$	$9\frac{5}{8}$
$5\frac{1}{4}$	$10\frac{3}{4}$	$11\frac{5}{6}$	$3\frac{7}{32}$

e.	f.	g.	h.
$10\frac{4}{5}$	$10\frac{1}{16}$	$6\frac{1}{3}$	$11\frac{3}{5}$
$7\frac{9}{20}$	$15\frac{3}{4}$	$7\frac{3}{4}$	$7\frac{11}{15}$
$10\frac{3}{4}$	$18\frac{5}{8}$	$9\frac{1}{2}$	$10\frac{2}{3}$

2. Perform the indicated operations. Express the sum in simplest form.

a. $10\frac{3}{5} + 14\frac{13}{20} + 7\frac{3}{4}$

b. $7\frac{2}{3} + 3\frac{1}{4} + 14\frac{5}{6}$

c. $16\frac{5}{7} + 9\frac{9}{14} + 7\frac{3}{28}$

d. $8\frac{9}{16} + 5\frac{3}{8} + 19\frac{5}{32}$

3. A carpenter needs three lengths of flooring measuring 10 3/16 in, 14 3/8 in., and 12 1/2 in. What length of flooring does he need to cut the three pieces? (Assume there is no waste.)

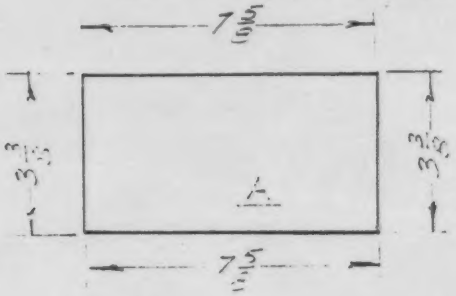
4. A salesperson sold the following cuts of silk material: 4 1/3 yards, 6 3/4 yards, and 2 1/2 yards. What was the total number of yards sold?

5. Mr. Watkins needs four lengths of pipe measuring 5 3/16 feet, 8 5/8 feet, 6 1/2 feet, and 11 3/4 feet. What is the total length of pipe needed to cut all four pieces?

6. Carmen bought 3/4 of a pound of cherries, 2 1/2 pounds of apples, and 2 3/8 pounds of pears. How many pounds of fruit did she buy?

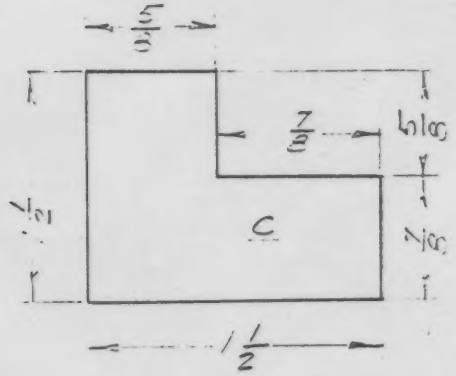
7. An electrician needs four lengths of 14-gauge electrical wire measuring 4 1/2 yards, 8 3/4 yards, 11 2/3 yards, and 7 5/6 yards. What is the total length of wire needed so that he can cut all four pieces?

8. Calculate the perimeter of each of the figures below. Place your answer next to the proper letters listed down the center of the sheet.



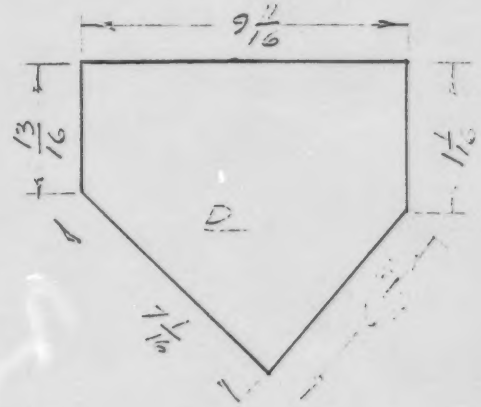
A. _____

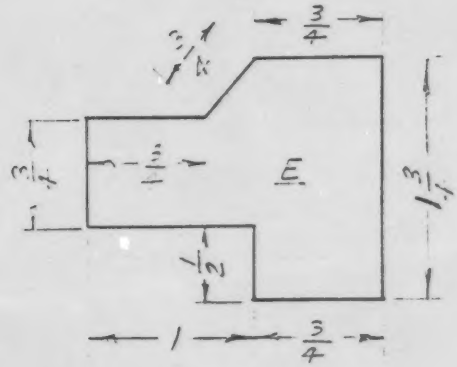
B. _____



C. _____

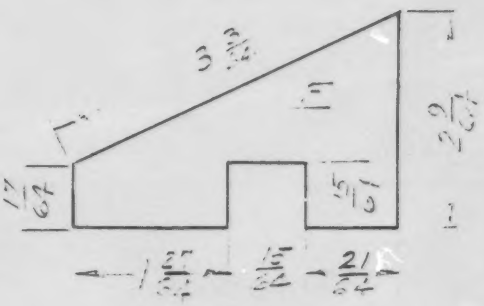
D. _____

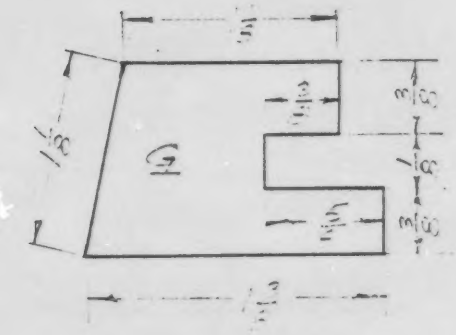




E. _____

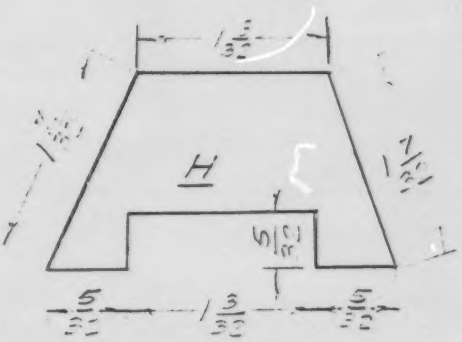
F. _____





G. _____

H. _____



Lesson 6

Aim

To subtract mixed numerals

Motivation

When full, a gasoline tank contains 18 1/2 gallons of gasoline. After filling up, the owner used 7 3/4 gallons on a trip. How can we find the number of gallons remaining in the tank?

Development

Teacher Activity	Student Activity
1. a) Elicit that the problem can be solved by subtraction.	1. a) Subtract: $18\frac{1}{2} = 18\frac{2}{4}$ $7\frac{3}{4} = 7\frac{3}{4}$
b) Have students realize that 3/4 is greater than 2/4.	
c) Just as in subtracting whole numbers, we must often exchange before we subtract.	c) Subtract: $\begin{array}{r} 72 \\ 46 \\ \hline \end{array}$ $\begin{array}{r} {\scriptstyle 6}\not{7}{\scriptstyle 1}\not{2} \\ 46 \\ \hline 26 \end{array}$
2. Develop the procedure for exchange with fractions: $18\frac{2}{4} = 18 + \frac{2}{4}$ $= 17 + 1 + \frac{2}{4}$ $= 17 + \frac{4}{4} + \frac{2}{4}$ $= 17 + \frac{6}{4}$ $= 17\frac{6}{4}$	2. Subtract: $18\frac{2}{4} = 17\frac{6}{4}$ $7\frac{3}{4} = 7\frac{3}{4}$ $10\frac{3}{4}$ There were 10 3/4 gallons left.
3. Have the students subtract: $\begin{array}{r} 12 \\ 5\frac{1}{4} \\ \hline \end{array}$	3. $12 = 11\frac{4}{4}$ $5\frac{1}{4} = 5\frac{1}{4}$ $6\frac{3}{4}$

Summary

1. When you have to find how much greater one number is than another number, what operation do you use?
2. What do we call the answer to a subtraction example?
3. To what whole number is $7\frac{3}{3}$ equivalent?

SUGGESTED EXERCISES

1. Subtract. Express each difference in lowest terms.

a.	b.	c.	d.
$15\frac{3}{4}$	$12\frac{7}{8}$	$5\frac{5}{8}$	$8\frac{11}{15}$
$5\frac{1}{2}$	$2\frac{5}{16}$	$3\frac{1}{6}$	$2\frac{2}{5}$
e.	f.	g.	h.
$23\frac{3}{8}$	$36\frac{13}{16}$	$57\frac{3}{5}$	$12\frac{3}{8}$
$14\frac{3}{4}$	$12\frac{7}{8}$	$29\frac{3}{4}$	$16\frac{3}{4}$

2. Perform the indicated operations. Express answers in lowest terms.

a. $(15\frac{7}{8}) + (7\frac{1}{2})$

b. $(19\frac{3}{4}) - (4\frac{2}{5})$

c. $(15\frac{3}{4}) - (7\frac{1}{2})$

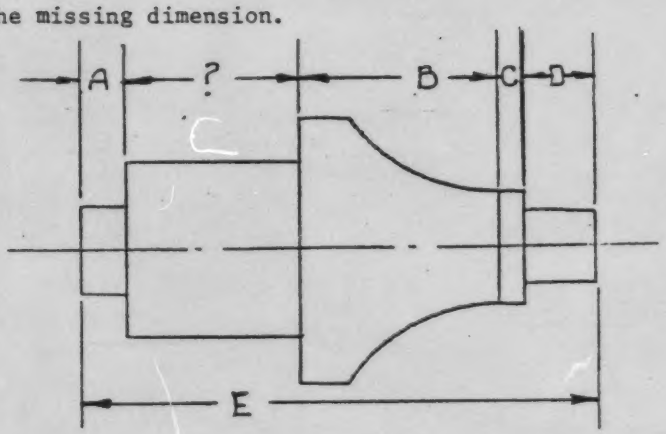
d. $(5\frac{3}{20}) + (6\frac{3}{10})$

e. $(15\frac{3}{10}) + (7\frac{7}{20}) - (3\frac{4}{5})$

f. $(19\frac{7}{8}) + (20\frac{1}{2}) - (9\frac{2}{3})$

3. Steve is a test driver who specializes in determining the fuel efficiency of different cars. In one series of tests, models A, B, and C averaged 26 3/8, 21 3/4, and 30 1/4 miles per gallon, respectively. How many more miles per gallon did C average compared with A ? How many fewer miles per gallon did B average compared with C ?

4. Find the missing dimension.



A = $5\frac{3}{4}$
B = $7\frac{2}{3}$
C = $1\frac{1}{2}$
D = $2\frac{3}{5}$
E = $21\frac{5}{6}$

5. Subtract. Express the differences in lowest terms.

a. $9\frac{3}{4}$ − $4\frac{3}{8}$ = _____

b. $9\frac{3}{4}$ − $6\frac{3}{8}$ = _____

c. $15\frac{2}{3}$ − $7\frac{3}{4}$ = _____

d. $9\frac{7}{16}$ − $4\frac{2}{3}$ = _____

e' $11\frac{1}{2}$ − $8\frac{1}{10}$ = _____

6. Perform the indicated operations. Express in lowest terms.

a. $(+7\frac{1}{2}) + (9)$

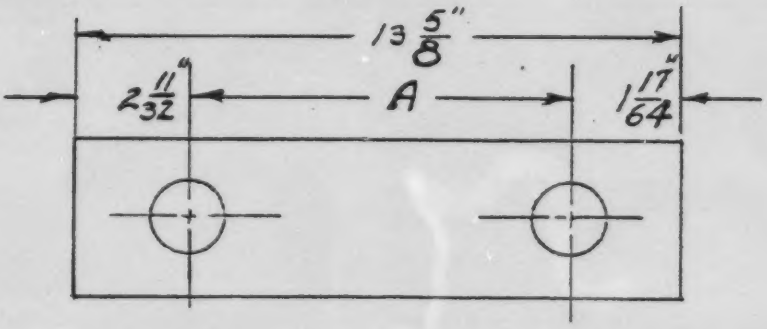
b. $(+8\frac{3}{4}) + (-11\frac{1}{2})$

c. $(+15\frac{7}{8}) + (+5\frac{11}{16})$

d. $(+18\frac{1}{2}) - (-11\frac{2}{3})$

e. $7\frac{1}{2} + 8\frac{1}{3} + 9\frac{3}{4} - 6\frac{3}{20}$

8. Find the value of Dimension A.



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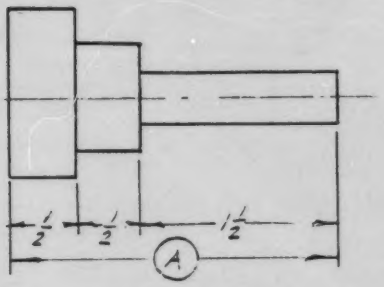
9. Calculate the missing dimensions. Place your answers next to the proper letters listed down the center of the sheet.

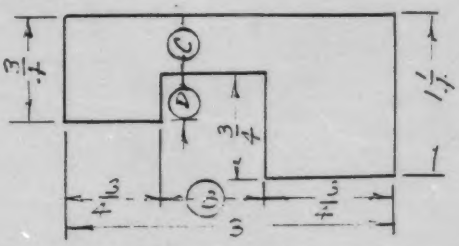


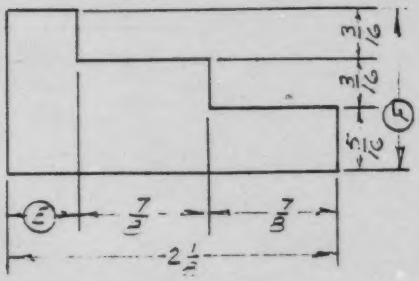
A____

B____

C____

D____



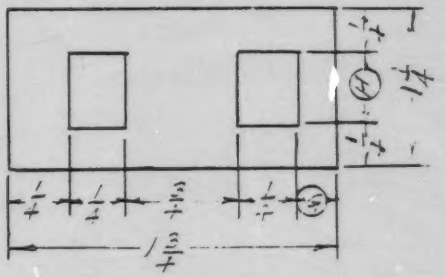


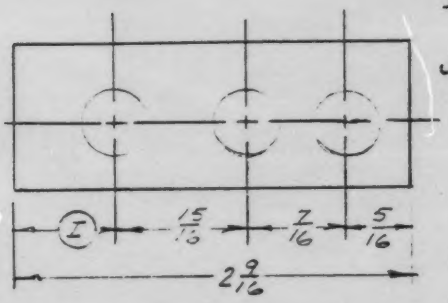
E____

F____

G____

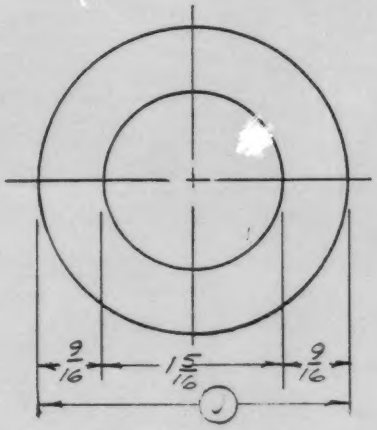
H____

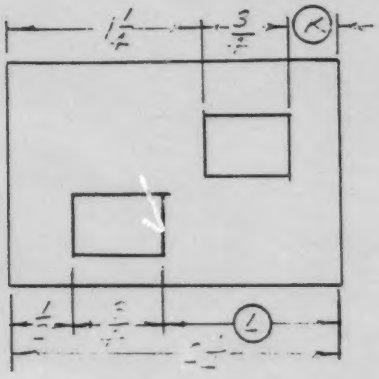




I____

J____



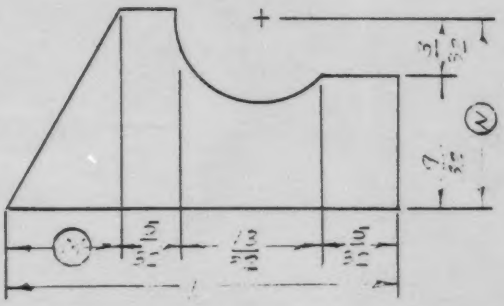


K____

L____

M____

N____



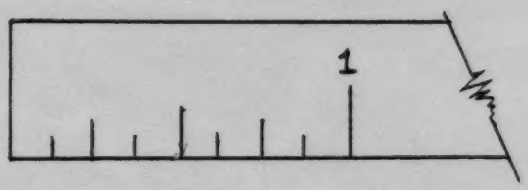
Lesson 7

Aim

To multiply fractions

Motivation

Draw the following diagram on the chalkboard. Pose the question:



Using the ruler, how can we determine the length of $\frac{1}{2}$ of $\frac{1}{4}$ inch?

Development

	Teacher Activity		Student Activity
1.	Since 1/2 of 1/4 = 1/8, and: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, have students conclude that the motivation can be solved by multiplication.	1.	Use the diagram to show that: $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$ To find a fraction of a number, we multiply.
2.	Have students generalize the rule for multiplication of fractions: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$	2.	We write the product of the numerators over the product of the denominators, and simplify, if possible.
3.	Find the product: a) $\frac{5}{6} \times \frac{1}{3}$ b) $\frac{2}{3} \times \frac{1}{4}$	3.	a) $\frac{5}{6} \times \frac{1}{3} = \frac{5 \times 1}{6 \times 3} = \frac{5}{18}$ b) $\frac{2}{3} \times \frac{1}{4} = \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{1}{6}$
4.	a) Multiply: $\frac{1}{2} \times 2\frac{1}{2}$ b) Elicit the procedure for converting a mixed numeral to an improper fraction. c) Use the improper fraction to multiply.	4.	b) $2\frac{1}{2} = \frac{5}{2}$ c) $\frac{1}{2} \times 2\frac{1}{2} = \frac{1}{2} \times \frac{5}{2} = \frac{1 \times 5}{2 \times 2} = \frac{5}{4} = 1\frac{1}{4}$

Summary

1. What is the product of 1/3 and 1/3 ?
2. What operation should we use to find 3/4 of 1/2 ?
3. When multiplying by a mixed numeral, what should we convert it to?

SUGGESTED EXERCISES

1. Find each of the following products:

a. $\frac{7}{8} \times \frac{3}{4}$ b. $\frac{3}{5} \times \frac{1}{2}$ c. $\frac{2}{3} \times \frac{3}{4}$ d. $\frac{5}{6} \times \frac{2}{5}$

e. $\frac{2}{3} \times 1\frac{1}{2}$ f. $\frac{5}{6} \times 1\frac{3}{5}$ g. $\frac{3}{4} \times 2\frac{1}{3}$ h. $\frac{2}{5} \times 1\frac{1}{4}$

i. $1\frac{3}{4} \times 1\frac{1}{7}$ j. $4\frac{1}{2} \times 2\frac{2}{3}$ k. $2\frac{2}{3} \times 2\frac{1}{4}$ l. $7\frac{1}{2} \times 1\frac{3}{5}$

2. If there are 7 1/2 gallons to the cubic foot, how many gallons of oil will a tank hold if it contains 9 1/3 cubic feet?
3. To find the circumference of a circle, multiply its diameter by 3 1/7. What is the circumference of a circle whose diameter is 3 1/2 inches?
4. Calculate the weight of 16 bars of cold rolled steel, each bar 9 3/4 feet long, if the steel weighs 2 1/4 pounds per foot.
5. A gasoline tank holds 22 gallons. How many miles is it possible to travel, on a tank full of gas, figuring 18 1/2 miles per gallon?
6. How long a piece of drill rod is required to make 12 drills, each 3 1/16 inches long, if 1/8 inch waste is allowed for each cut?
7. Grease is bought in a barrel containing approximately 54 gallons. How many pounds of grease should it contain? Use 7 7/8 pounds per gallon.
8. A repairman cut 12 pieces of copper from a coil. Each piece was 27 1/16 inches long. What was the total length of copper used?

Lesson 8

Aim

To divide fractions and mixed numerals

Motivation

What number, when multiplied by 5/6, equals one?

Development

Teacher Activity	Student Activity
1. a) Elicit the solution to the motivation problem.	1. a) $\frac{5}{6} \times \frac{6}{5} = \frac{30}{30} = 1$
b) Inform students that 5/6 and 6/5 are <u>reciprocals</u> of each other. Define reciprocals as two numbers whose product is one. (Another term for reciprocal is multiplicative inverse.)	
2. Find the reciprocal of each: a) $\frac{7}{10}$ b) $\frac{9}{4}$ c) 7 d) $1\frac{2}{3}$ Recall that $7 = \frac{7}{1}$ and $1\frac{2}{3} = \frac{5}{3}$	2. a) $\frac{10}{7}$ b) $\frac{4}{9}$ c) $\frac{1}{7}$ d) $\frac{3}{5}$
3. Have students solve the following: a) $8 \div 2$ b) $8 \times \frac{1}{2}$	3. a) $8 \div 2 = 4$ b) $8 \times \frac{1}{2} = \frac{8}{1} \times \frac{1}{2} = \frac{8}{2} = 4$
4. a) Elicit that both examples had the same answer.	4. a) Dividing by 2 gives the same answer as multiplying by ½.
b) Have students generalize: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	b) Dividing by a number is equivalent to multiplying by its reciprocal.
5. Divide: $4\frac{3}{8} \div 1\frac{1}{4}$	5. $4\frac{3}{8} \div 1\frac{1}{4} = \frac{35}{8} \div \frac{5}{4} = \frac{35}{8} \times \frac{4}{5} = \frac{140}{40} = 3\frac{20}{40} = 3\frac{1}{2}$

Summary

1. When we multiply a number by its reciprocal, what is the product?
2. What is the reciprocal of 4/5 ? of 8?
3. Dividing by 2/3 is equivalent to multiplying by what number?
4. How do we divide by a fraction or mixed numeral?

SUGGESTED EXERCISES

1. Divide:

a. $\frac{3}{6} \div \frac{2}{4}$ b. $\frac{5}{8} \div \frac{1}{2}$ c. $\frac{2}{5} \div \frac{4}{5}$ d. $\frac{1}{3} \div \frac{1}{2}$

e. $\frac{2}{3} \div \frac{4}{5}$ f. $\frac{5}{8} \div \frac{5}{8}$ g. $\frac{1}{2} \div \frac{7}{16}$ h. $\frac{7}{8} \div \frac{3}{8}$

i. $13\frac{1}{4} \div 2\frac{1}{4}$ j. $15\frac{5}{8} \div 6\frac{1}{4}$ k. $4\frac{1}{4} \div \frac{1}{8}$ l. $18 \div 2\frac{1}{4}$

2. To make a six-sided cabinet, a piece of mahagony 9 3/4 feet long must be sawed so that all sides are equal. What will the length of each piece be?
3. A strip of copper 26 1/4 inches long is to be sheared into six equal pieces. What is the length of each piece?
4. How many siding shingles, each 13/16 inch thick, are there in a stack 2 feet 2 inches high?
5. If a board 10 1/4 yards long is cut into four equal pieces, how long will each piece be?
6. How many 1/10's of a pound are there in 1/5 pound?
7. Juan drove 154 miles in 3 1/2 hours. How many miles did he average in one hour?
8. How many pieces of copper pipe 2 1/2 inches long can be cut from a piece 15 feet 6 inches long?
9. A 50 pound bag of peanuts is to be repackaged into small bags holding 5/16 of a pound each. How many smaller bags can be filled?

UNIT III

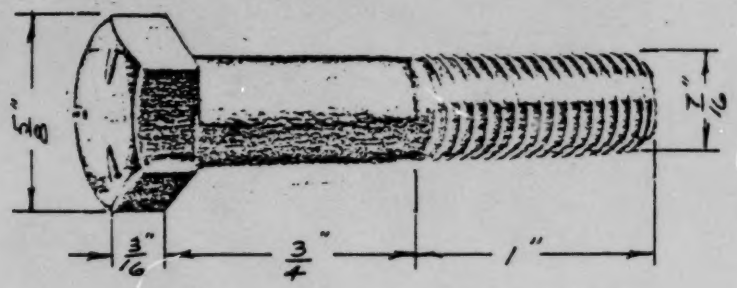
Lesson 9

Aim

To convert a fraction to a decimal and to read decimal equivalent

Motivation

The dimensions of the following bolt are given in fractions. To machine this bolt on a metal lathe that is calibrated in thousandths of an inch, it is necessary to change all dimensions in fractional form to their decimal equivalents.



How can each of the fractions in the diagram above be changed into its decimal equivalent?

Development

Teacher Activity	Student Activity
1. Elicit the different methods of converting fractions to decimal form.	1. Possible student responses: a. We can measure the shank with a ruler calibrated in decimals. b. We can look for the decimal equivalent on a conversion table. c. We can mathematically convert a fraction to its decimal equivalent.

2. Recall that a fraction may be interpreted as the division of two numbers.

$\frac{4}{4} = 1 \quad \frac{5}{4} = 1\frac{1}{4} \quad \frac{8}{4} = 2$

2. $\frac{3}{4} = 3 \div 4$ or $4\overline{)3}$

$\frac{5}{8} = 5 \div 8$ or $8\overline{)5}$

3. Refer to experiences with money where:
$ 5 = $ 5.00
$ 36 = $ 36.00

By placing a decimal point followed by zeros after the whole numbers, the value is not changed, but division becomes impossible.

3. $\frac{3}{4} = 4\overline{)3.00} = .75$

$\frac{5}{8} = 8\overline{)5.000} = .625$

4. Have the students change the remaining fractions in the motivation to decimal form.

4. $\frac{3}{16} = .1875$

$\frac{7}{16} = .4375$

5. Change each to an equivalent decimal:

a) $\frac{7}{10}$ b) $\frac{31}{100}$

5. a) $\frac{7}{10} = 10\overline{)7.0} = .7$

b) $\frac{31}{100} = 100\overline{)31.00} = .31$

6. Review the place value and reading of decimal numerals.

$\frac{7}{10} = .7$ = seven tenths

$\frac{31}{100} = .31$ = thirty-one hundredths

$\frac{625}{1000} = .625$ = six hundred twenty-five thousandths

6. Students will add the names of each place value to a place value chart:

5	.	9	6	7	3
whole numbers		tenths	hundredths	thousandths	ten-thousandths

Summary

1. How can we change a fraction into an equivalent decimal?
2. Explain how you can tell whether a decimal numeral is expressed in tenths, hundredths, or thousandths.
3. How is the fraction one-half expressed as a decimal?

SUGGESTED EXERCISES

1. Find the decimal equivalents for each of the following:

a) $\frac{1}{4}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$ d) $\frac{21}{32}$

e) $\frac{7}{32}$ f) $\frac{11}{16}$ g) $\frac{63}{64}$ h) $\frac{9}{16}$

i) $\frac{15}{20}$ j) $\frac{30}{48}$ k) $\frac{18}{24}$ l) $\frac{16}{32}$

2. Express each of these decimal numerals in words:

a) .8 b) .1 c) .45 d) .87

e) .775 f) .03 g) .009 h) .104

i) 3.2 j) 16.84 k) 44.440 l) 9.002

3. Complete the following chart:

WORD STATEMENTS	FRACTION	DECIMAL EQUIVALENT
eight tenths		
five hundredths		
forty-seven thousandths		
twenty-three hundredths		
one eighth		
two thirds		
eight twelfths		
three fifths		
fourteen twentieths		
sixteen sixteenths		

Lesson 10

Aim

To convert a decimal to a fraction

Motivation

An auto-parts supply book lists the dimensions of all parts in fractions. What size piston rings must a mechanic buy to replace rings that are 3.875 inches in diameter?

Development

Teacher Activity	Student Activity
1. Many dimensions are stated as decimals. At times it is necessary to convert these decimals to fractions. How can the fractional equivalent form of 3.875 be determined?	1. Possible student responses: a. We can measure the piston rings with a ruler calibrated in fractions. b. We can look for fractional equivalents on conversion tables. c. We can mathematically convert decimals to fractions.
2. Have the students recall the word statements and fractional equivalents for basic decimals. Emphasize that, when reading mixed decimal numerals, the location of the decimal point is indicated by the word "and".	2.
3. Have students read 3.875, express it as a fraction, then simplify it.	3. Three and eight hundred seventy-five thousandths. $3.875 = 3\frac{875}{1000} = 3\frac{7}{8}$
4. Have students list or memorize the most important decimal- fraction equivalents for ready reference and use.	4. $.5 = \frac{1}{2}$ $.25 = \frac{1}{4}$ $.125 = \frac{1}{8}$ $.75 = \frac{3}{4}$ $.375 = \frac{3}{8}$ $.625 = \frac{5}{8}$ $.875 = \frac{7}{8}$

Decimal	Read as:	Fraction
.1	one tenth	$\frac{1}{10}$
.01	one hundredth	$\frac{1}{100}$
.99	ninety-nine hundredths	$\frac{99}{100}$
.001	one thousandth	$\frac{1}{1000}$
4.9	four and nine tenths	$4\frac{9}{10}$

Summary

1. How can we write the decimal,"point fifteen", as a fraction?
2. What is the value of "two point five", expressed as a mixed numeral?
3. How is "nine thousandths" written as a decimal?

SUGGESTED EXERCISES

1. Write each decimal as a mixed numeral or fraction in lowest terms.

a. .435	b. .280	c. .62
d. .2 m	e. .1875 in.	f. 1.25 ft
g. .0937	h. .4	i. 1.3125
j. 2.75 lbs	k. .0375 in.	l. .875 lb

2. Complete the following chart:

DECIMAL	WORD STATEMENT	FRACTION
.1		
.29		
.482		
	six hundredths	
	eight tenths	
.096		
.140		
.075		

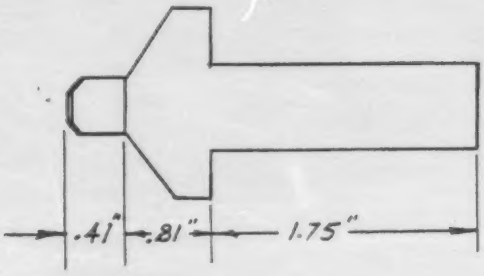
UNIT III

Lesson 11

Aim

To add and subtract decimals

Motivation



TAPER PIN

Using the taper pin illustrated in the diagram above, find its total length.

Development

Teacher Activity	Student Activity
1. Elicit that the problem in the motivation requires the operation of addition.	
a. Have students use their previous knowledge of how to convert decimals to fractions to solve the motivation problem:	a. $.41 + .81 + 1.75 =$ $\frac{41}{100} + \frac{81}{100} + \frac{175}{100} = \frac{297}{100} = 2.97$
b. Remind students that when the vertical form is used, the decimal points must be aligned. This enables tenths to be added to tenths, hundredths to hundredths, etc.	b. .41 .81 1.75 sum: 2.97

2. a) Have the students add:

3.783 + 0.469

or

6.37 + 4.6 + .573 + 2

b) Since 10 tenths equals one whole, we exchange 'across' the decimal point.

2.

$$\begin{array}{r} 6.370 \\ 4.600 \\ 0.573 \\ 2.000 \\ \hline 13.543 \end{array}$$

3. Develop the subtraction of decimals in the same manner.

3.

a) Subtract: $\begin{array}{r} .783 \\ .412 \\ \hline \end{array}$

a) $\begin{array}{r} .783 \\ .412 \\ \hline .371 \end{array}$ (no exchange)

b) Subtract: $\begin{array}{r} 5.317 \\ 0.546 \\ \hline \end{array}$

b) $\begin{array}{r} 5.317 \\ 0.546 \\ \hline 4.771 \end{array}$ (with exchange)

c) Subtract: 4 - 0.25

c) $\begin{array}{r} 4.00 \\ 0.25 \\ \hline 3.75 \end{array}$ (4 = 4.00)

Summary

1. What is the place value of the first digit to the right of the decimal point?

2. How do we write the numerals when adding or subtracting decimals?

SUGGESTED EXERCISES

1. Add:

a 5.21 + 4.4 + .219 + 5

b 11 + 1.02 + .003 + 7.1

c 3.002 + 7 = .7 = 7.23

d	e
.334	.0063
.628	.0872
.792	.0580
.873	.0105
.746	.2037
.489	.3078

1 (cont.) Add:

f	g	h
.782	.0346	9.230
.626	.0753	7.204
.542	.0634	3.409
.567	.0438	5.076
.112	.0326	2.584

i Four flat metal pieces are stacked on top of each other. If the thicknesses of the pieces are 3 in., .002 in., 4.92 in., and .85 in., what is the height of the stack?

j Five rods are laid end to end in a straight line. The lengths of the rods are 3.21 in., 5 in., 7.8 in., .725 in., and .9 in. What is the total length of the rods

2. Subract:

a	b	c	d
6.2500	4.8750	.84375	.65625
3.0625	2.5625	.71875	.18750

e
.3707
.2954

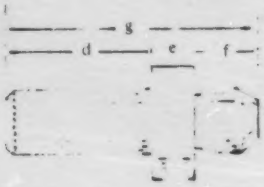
f 5.7 -.481

g 13 - 8.05

h 2.407 - 1.828

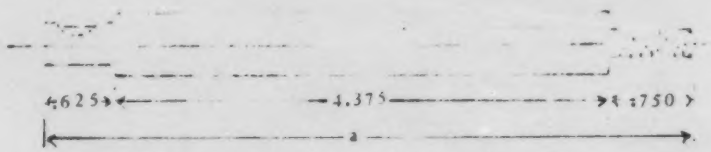
3. Find the total length g of the following standard locator pins, for which the dimensions d, e, and f are:

Pin	d	e	f
#241	.6250	.2500	.3125
#245	1.1250	.5000	.4375
#252	1.2500	.4375	.5750
#258	1.7500	.5625	.5000



4. If the small diameter of a tapered pin is 0.15625" and the large diameter is 0.06250" more, what is the large diameter of the pin?

5. Find the length a of the shaft represented by the drawing below:



6. The length of a piece of tubing for use on an airplane is to be 4.487". However, a piece that is not more than .002" longer will be accepted. What is the greatest length that will be accepted?

UNIT III

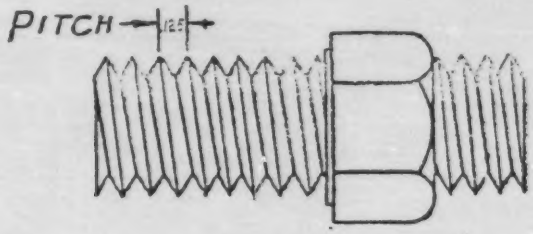
Lesson 12

Aim

To multiply decimals

Motivation

In the diagram below, the pitch of a thread is .125". How far will the nut move when 6 turns are made?



Development

Teacher Activity	Student Activity
1. Discuss the problem in the motivation. Elicit that the problem requires multiplication.	1. Student answers - the nut moves .125" in one turn. Therefore 6 x .125 in 6 turns.
2. a. How can .125 be written as a fraction?	a. $.125 = \frac{125}{1000}$
b. Multiply $\frac{125}{1000} \times 6$.	b. $\frac{125}{1000} \times 6 = \frac{750}{1000}$
c. How is the result expressed as a decimal?	c. .750
d. Recall that the multiplication could have been done vertically, using decimals.	d. $\begin{array}{r} .125 \\ \times \quad 6 \\ \hline .750 \end{array}$
e. Elicit the need for a rule to locate the decimal point.	

3. Have the students convert the following to fractions, multiply, and then express the product as a decimal.
   a) .3 x .2
   b) .7 x .25
   c) .5 x 6

3. a) $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100}$ or .06

   b) $\frac{7}{10} \times \frac{25}{100} = \frac{175}{1000}$ or .175

   c) $\frac{5}{10} \times \frac{6}{1} = \frac{30}{10}$ or 3

4. By having the examples written in vertical form help the students discover that the number of decimal places in the product is equal to the sum of the number of decimal places in the two factors.

4. a) $\begin{array}{r} .3 \\ \times\ .2 \\ \hline .06 \end{array}$ b) $\begin{array}{r} .25 \\ \times\ .7 \\ \hline .175 \end{array}$

   c) $\begin{array}{r} .5 \\ \times\ 6 \\ \hline 3.0 \end{array}$

5. Have the students first state the number of decimal places there will be in the product, and then multiply:
   a) .15 x .3
   b) .04 x .05
   c) 12.5 x .5

5.

a)	Three.	.045
b)	Four.	.0020
c)	Two.	6.25

Summary

1. What is the product of one tenth and one tenth?
2. When we multiply .35 by .45, how many decimal places should there be in the product?
3. When multiplying decimals, how do we locate the decimal point in the product?

SUGGESTED EXERCISES

1. a. 25 x 25 b. .25 x 2.5
   c. 2.5 x 25 d. .25 x .25
   e. 2.5 x 2.5 f. .25 x 25

2. Multiply the following:

a.	2 x 1.5	b.	1.23 x .71
c.	1.5 x 3	d.	1.1 x .11
e.	4.5 x 7.5	f.	.148 x .125
g.	12.4 x 7.5	h.	.1875 x 16
i.	1.1 x 1.1	j.	.625 x .5
k.	.2 x .15	l.	.15 x .3
m.	.875 x 2.4	n.	.75 x .24

3. A three-cell storage battery, on discharge, gave a voltage of 1.85 volts per cell. What was the total voltage of the battery?

4. A test showed that a fully charged storage battery was discharged by a current of 11.25 amps in 7.2 hours. What is the probable capacity of this battery expressed in ampere-hours? (The number of ampere-hours equals the product of the ampere discharge rate and the number of hours necessary to discharge the battery.)

5. The width of a hexagon nut, across the corners, is equal to the product of 1.7321 and the diameter of the bolt. Find the width of the hexagon nut for a bolt whose diameter is .75 inches. Round your answer to the nearest hundredth.

6. If a six-cell storage battery showed an average voltage of 2.2 volts per cell, what was the battery's voltage?

7. The number of pounds of water in a tank is equal to the volume in cubic feet multiplied by 62.425. What is the weight of the water in a 6.4 cubic foot tank?

8. Three popular models of automobiles have these specifications:

	Displacement per Cylinder	Number of Cylinders
a.	.35 liters	4
b.	.41 liters	6
c.	.37 liters	8

Find the total engine displacement for each model.

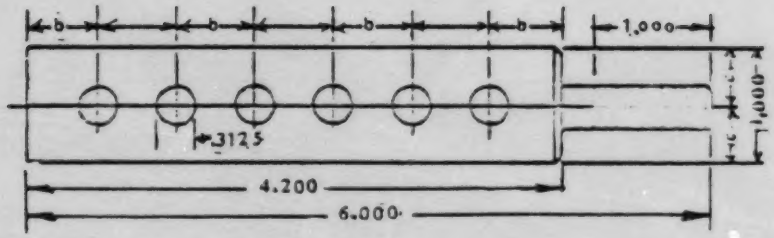
Lesson 13

Aim

To divide decimal numerals

Motivation

Find the dimension represented by the letter b in the diagram.



Development

Teacher Activity	Student Activity
1. a) Elicit that the length 4.200 or 4.2 is to be divided into seven equal parts. This requires the use of division.	1. a) 4.200 = 4.2 4.2 ÷ 7 or $\frac{4.2}{7}$
b) The decimal point in the dividend is "brought up" and placed in the quotient.	b) $7\overline{)4.2}$ .6 quotient dividend Check: What is .6 x 7 ? .6 x 7 = 4.2 Dimension b is 0.6 units.
2. a) Pose the problem: How many washers, at $.50 each can be bought for $3.50?	2. a) $.50\overline{)\$3.50}$?
b) Elicit that the answer is "seven washers". But just bringing up the decimal point will not give the correct answer.	

3. Recall that:
.2 x 10 = 2.
.15 x 10 = 1.5
.15 x 100 = 15.
.175 x 100 = 17.5
.175 x 1000 = 175.

3. To multiply a number by a power of ten, move its decimal point to the right the same number of places as the number of zeros in the power of ten.

4. By what number can we multiply the numerator and denominator of

$$\frac{3.50}{.50}$$

to eliminate the decimal point from the divisor?

4. Multiply them by 100.

$$\frac{3.50}{.50} \times \frac{100}{100} = \frac{350}{50} = 7$$

5. Develop the algorithm for division by a decimal numeral.

5.
$$.50 \overline{)\,3.50} \quad 7.$$

6. Divide: $.25 \overline{)\,4}$

6.
$$.25 \overline{)\,4.00} \quad 16.$$

Summary

1. How can we multiply a number by a power of ten?

2. In the example 6 ÷ 3 = 2, which number is the divisor? What is the quotient?

3. When we divide a decimal numeral by a whole number, what do we do with the decimal point?

4. When there is a decimal point in the divisor, what do we do before dividing?

SUGGESTED EXERCISES

1. Divide:

a. 12 ÷ 1.5
b. 1.2 ÷ 15
c. 3.9 ÷ 3
d. 3.9 ÷ .3
e. 39 ÷ .03
f. .122 ÷ .002
g. .0021 ÷ .007
h. 1932 ÷ 2.1
i. 1.462 ÷ 3.4
j. 146.2 ÷ .043

2. A used car dealer sold 34 automobiles and the total sales amounted to $26,690. What was the average price of each car?

3. The piston displacement of a certain six-cylinder engine is 452.4 Cubic inches. What is the displacement of one cylinder?

4. A pail of lubricating grease contains 42.5 pounds and costs $6.80. What is the cost per pound?

5. A number of pieces of radiator hose totaling 42 feet cost $7.35. What is the cost per foot?

6. The cost of 22 wrench sets was $319. What was the cost per set?

7. A length of a steel frame is 14.85 inches. If this length is to be divided into four equal parts, how long will each part be?

8. A man is paid $21.33 for a 135.5 mile trip in his automobile. What is he charged per mile (to the nearest cent)?

9. A car owner's repair bill came to $85.80. He paid one-half of the bill in cash, and the remainder in five equal payments. What was the amount of each of the payments?

10. If a van ran 643.5 miles on 45 gallons of gasoline, how many miles per gallon did it average?

11. On a certain trip, an automobile traveled at an average rate of 46.5 miles per hour. By the end of the trip it had covered 279 miles. How much time was required to make the trip?

12. A truck driver who charges $1.20 per parcel for delivery earned $54 in one day. How many parcels did he deliver?

13. What is the average speed, in miles per hour, of a car that is driven 372 miles in 7.5 hours?

14. A workman's pay check was $246 per week of six days. What was his rate of pay, per hour, figuring 8.2 hours per day?

15. If 15 gallons of gasoline weigh 98.4 pounds, what will one gallon weigh?

16. A tire cost $56.92 and was used on a car for 28,460 miles. What was the cost per mile?

17. The cost of maintaining a fleet of 12 small trucks for a year was $11,232. What was the average cost per week for one truck?

18. A man rents his truck for $35.20 per day. What is the rental rate per hour, on the basis of an eight-hour day?

Lesson 14

Aim

To relate fractions, decimals, and percents

Motivation

Tom earned $100. He spent $75 of it for a new set of tools. What are some of the ways you can describe what part of his earnings was spent for the tools?

Development

Teacher Activity	Student Activity
1. a) Discuss the motivation.	1. a) Tom spent 75/100 or 3/4 of his earnings for tools.
b) Point out that .75 of his earnings could be misinterpreted to mean 75¢ of his earnings.	b) Tom spent 75/100 or 0.75 of his earnings for tools.
c) Elicit that a comparison or ratio is frequently expressed as a percent.	c) Tom spent 75/100 or 75% of his earnings for tools.
2. a) Since percent means "hundredths" or "per hundred", a percent can be changed to a fraction by writing it over 100 and simplifying.	2. a) $75\% = \frac{75}{100} = \frac{3}{4}$
3. Have the students change these percents to fractions.	3. a) $50\% = \frac{50}{100} = \frac{1}{2}$
a) 50%	b) $25\% = \frac{25}{100} = \frac{1}{4}$
b) 25%	c) $17\% = \frac{17}{100}$
c) 17%	d) $100\% = \frac{100}{100} = 1$ (all of a thing)
d) 100%	e) $150\% = \frac{150}{100} = 1\frac{1}{2}$
e) 150%	
4. Elicit that hundredths can be expressed by a two place decimal. Have students conclude that a percent can be changed to a decimal by moving the decimal point two places to the left.	4. $75\% = \frac{75}{100} = .75$

5. Have students change these percents to decimals:

   a) 35%
   b) 7%
   c) 125%
   d) 0.5%

5.

   a) 35% = .35 or 0.35
   b) 7% = .07 or 0.07
   c) 125% = 1.25
   d) 0.5% = 0.005

6. Pose the question: If we are given a decimal, how can we change it to a percent?

6. We can move the decimal point two places to the right, and write a percent sign.

7. Have the students change these decimals to percents:

   a) .63
   b) .05
   c) .125
   d) 2.50

7.

   a) .63 = 63%
   b) .05 = 5%
   c) .125 = 12.5%
   d) 2.50 = 250%

8. Which of these fractions can you convert to a percent?

   a) 37/100
   b) 9/100
   c) 113/100
   d) 3/50
   e) 1/4

8.

   a) 37/100 = 37%
   b) 9/100 = 9%
   c) 113/100 = 113%
   d) $\frac{3}{50} = \frac{6}{100} = 6\%$
   e) $\frac{1}{4} = \frac{25}{100} = 25\%$

9. When can we readily change a fraction to a percent?

9. A fraction can be changed to a percent easily if the denominator is 100 or can readily be changed to 100.

Summary

1. What does the term "percent" mean?
2. How can we change a percent to a fraction?
3. How can we change a percent to a decimal?
4. How can we change a decimal to a percent?
5. Which is greater: 21%, 21/100, 0.21 ?

SUGGESTED EXERCISES

1. Change each percent to a decimal and then to a fraction in lowest terms.

a.	39%	27%	83%
b.	80%	40%	60%
c.	35%	85%	55%
d.	7%	3%	9%
e.	48%	82%	36%
f.	91%	1%	101%
g.	200%	150%	175%
h.	15%	1.5%	0.5%

2. Change each decimal to an equivalent percent.

a.	0.40	0.36	0.19
b.	0.04	0.84	0.73
c.	1.25	1.15	2.75
d.	0.175	0.375	0.425

3. Change each fraction to an equivalent percent.

a.	$\frac{33}{100}$	$\frac{97}{100}$	$\frac{7}{100}$
b.	$\frac{9}{10}$	$\frac{7}{20}$	$\frac{19}{100}$
c.	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{13}{50}$

4. Last Tuesday, all 28 students in Kathy's class were present. What percent of the class was present?

5. On a test with 100 questions, John had 73 correct. What percent was his mark?

6. May has twice as many stamps in her collection as Ken does. What percent of Ken's stamps does May have?

Lesson 15

Aim

To find a percent of a number

Motivation

On a woodworking project, Alice planned to allow 20% for possible waste. If she purchased a 10 foot board, how much did she allow for waste?

Development

Teacher Activity	Student Activity
1. a) Elicit that we have to find 20% of the 10 feet.	1.
b) What does 20% mean?	b) $20\% = \frac{20}{100}$ or $\frac{1}{5}$ $20\% = .20$ or 0.20
c) What is 1/5 of 10?	c) $\frac{1}{5}$ of $10 = \frac{1}{5} \times \frac{10}{1} = \frac{10}{5} = 2$
d) What is 0.20 of 10?	d) $0.20 \times 10 = 2.00$ or 2
e) What did Alice allow for waste?	e) She allowed two feet for waste.
2. How were we able to find a percent of the number?	2. To find a percent of a number, change the percent to an equivalent decimal or fraction, and multiply.
3. Pose the problem: The Velez family is taking a 1425 mile trip. They plan to drive about 25% of the distance each day. To the nearest mile, how far do they have to drive each day? (Review rounding numbers.)	3. $25\% = 0.25 = 1/4$ $1425 \times 0.25 = 7125 + 2850 = 356.25$ $\frac{1}{4} \times \frac{1425}{1} = \frac{1425}{4} = 356\frac{1}{4}$ They should drive a little more than 356 miles per day.

4. Review rounding with dollars and cents.

a) $2.1925

b) $5.9975

4.

a) $2.1925 rounds to $2.19

b) $5.9975 rounds to $6

5. Pose the problem: If the tax rate is 8%, what is the sales tax on a purchase of:

a) $35

b) $17.70

5. a)

$ 35
x .08
$ 2.80

The tax is $2.80.

b)

$ 17.70
x .08
$ 1.4160

The tax is $1.42.

Summary

1. How do we find a percent of a number?

2. What is 100% of $15.75 ?

SUGGESTED EXERCISES

1. a. Find 45% of $400.

b. Find 62% of $300.

c. What is 9% of 750?

d. What is 71% of 85?

e. Find 25% of 8400 lbs.

f. Find 20% of 200 minutes.

g. What is 85% of 40 in.?

h. What is 37% of 400 meters?

2. A bicycle sells for $152. If the sales tax rate is 8%, what is the sales tax? How much does the purchaser have to pay for the bicycle?

3. The Federal excise tax on jewelry is 20%. The local sales tax is 8%. How much must one pay for a watch that lists for $188?

4. If the mechanical efficiency of an automobile is 85% in low gear, find the brake horsepower (the horsepower delivered to rear wheels) when the indicated horsepower is 86 hp.

5. An airplane engine consumes 1800 liters of fuel and air per hour. If 7.5% of this mixture is fuel, how many liters of fuel are required per hour?

6. If the true values of certain resistors are within 5% of rate values, what is the maximum resistance that could be expected from a resistor marked 300 ohms?

7. A thermometer reads 98° Celsius. In this range it is known to read 3% higher than true temperature. What is the true temperature?

8. The most effective germicidal strength of alcohol is a 70% solution. How many ounces of alcohol would be required to make a 12 ounce bottle of 70% alcohol?

9. A quantity of brass weighing 3200 pounds consisted of 61% copper, 36% zinc, and 3% lead. How many pounds of each metal was contained in the brass?

10. A set of pulleys required a force of 45 pounds to lift a car. After oiling the pulleys, the required force was reduced 20%. What force was then needed to lift the car?

11. A cubic foot of ice weighs 91.7% as much as a cubic foot of water. If water weighs 62.425 pounds per cubic foot, what is the weight of a cubic foot of ice?

12. The ABD Manufacturing Corporation agreed to give its employees a 12% salary increase. If Martha's salary had been $225 per week, what was her new salary?

13. A tire selling for $55 is claimed to give 150% as much mileage as one selling for $44. If the $44 tire gives an estimated mileage of 20,000 miles, how many miles can be expected from the $55 tire?

14. A cast iron pulley weighed 136 pounds. During machining, 9% of the weight was lost. What was the weight of the pulley after machining?

15. In a batch of concrete, 16 2/3% of the weight was cement, 33 1/3% was sand, and 50% was gravel. If the batch weighed 1800 pounds, how many pounds of each material were used?

Lesson 16

Aim

To find what percent one number is of another

Motivation

Carl bought 1000 feet of cable to rewire an old building. He began in the basement and used 280 feet of cable there. What part of the wire did he use in the basement? What percent is this?

Development

Teacher Activity	Student Activity
1. a) Have the students express the part used as a fraction.	1. a) He used $\frac{280}{1000}$ of the cable.
b) Elicit that percent is a ratio or comparison based upon 100.	b) $\frac{280}{1000} = \frac{28}{100} = 28\%$ Carl used 28% of the cable.
2. Have the students state the procedure (or steps) just used to find what percent one number is of another.	2. a) Express the relationship as a fraction. b) Change the fraction to hundredths. c) Convert to a percent.
3. Pose the problem: Out of 225 tons of scrap metal collected, 54 tons was aluminum. What percent of the scrap was aluminum?	3. a) $\frac{54}{225}$ was aluminum. b) $\frac{54}{225} = 225 \overline{)54.00}$ (0.24) c) 0.24 = 24% aluminum.
4. Pose the problem: Erika earns $420 per week, compared to Bob's $280. Erika's salary is what percent of Bob's?	4. a) $\frac{420}{280}$ (Erika's salary compared to Bob's) b) $280 \overline{)420.00}$ (1.50) c) 1.50 = 150% Erika earns 150% as much as Bob.

Summary

1. Seven is what fractional part of 100?
   Seven is what percent of 100?

2. How do we find what percent one number is of another?

SUGGESTED EXERCISES

1. a. 12 is what percent of 24?

   b. 12 is what percent of 60?

   c. 15 is what percent of 60?

   d. 36 is what percent of 108?

   e. 36 is what percent of 72?

   f. 36 is what percent of 36?

   g. 36 is what percent of 18?

   h. 21 is what percent of 56?

2. Six quarts of Prestone are required to protect a 14 quart cooling system from freezing. What is the percent of Prestone in the cooling system?

3. A mixture of sugar and water contains 12 pounds of water and 8 pounds of sugar. What percent of the mixture is sugar?

4. Electrolyte for a car battery is prepared by adding 4 parts of water to one part of acid. What percent of the mixture is water, and what percent is acid?

5. A varnish contains 3 parts of asphaltune, 4 parts of boiled linseed oil, and 18 parts of turpentine. Rewrite this information using percents instead of parts.

6. If 25 ml of pure boric acid is dissolved in 975 ml of water, what is the percent strength of the solution?

7. If 10 ounces of lysol are added to 390 ounces of water, what is the percent strength of the lysol solution?

8. In making a soap, a chemist uses 12 quarts of fat, 12 quarts of water, and 1 pint of lye. Express this information using percents.

Unit IV

STATISTICS AND PROBABILITY

Lesson 1	To collect and organize data (GHSM p. 47)*
Lesson 2	To select a random sample (GHSM pp. 45-49)*
Lesson 3	To use the mean to interpret data (GHSM pp. 29-31, 47)*
Lesson 4	To use the mode to interpret data
Lesson 5	To use the median to interpret data (GHSM p. 47)*
Lesson 6	To define and use the concept of a sample space (GHSM pp. 45-47)*
Lesson 7	To develop the concept of probability (GHSM p. 43, pp. 45-47)
Lesson 8	To determine probabilities through experiments and by using addition (GHSM pp. 22-23, 43, 45-47)*
Lesson 9	To compute probabilities involving multiplication (GHSM pp. 24-25, 43, 45-47)*

*General High School Mathematics, New York State Department of Education, August 1978.

UNIT IV

STATISTICS AND PROBABILITY

Lesson 1

Aim

To collect and organize data

Motivation

The tenth-year math class is having a party. How may the most popular dessert be selected for the party?

Materials

Prepared Class Survey Data Form

Development

Teacher Activity	Student Activity
1. Using the problem in the motivation, elicit the need to collect information on popular desserts. Have students suggest the need: a. To list or collect data (collected information) b. To tally given data c. To interpret the given data	1. Hand out the Prepared Class Survey Data Form for popular desserts. Note: Additional space has been allotted for desserts not listed.
2. Have students complete the Class Survey Data Form (at the end of the lesson), filling out the appropriate columns. See sample to the right. a. Review how to change fractions to percents. b. Total columns, Number and Percent c. Show that the sum total for the Ratio column is one.	

FAVORITE DESSERTS	TALLY	NUMBER	RATIO	PERCENT
BROWNIES	|||	3	$\frac{3}{30}$	10%
CAKE		0	$\frac{0}{30}$	0%
CANDY	$\cancel{\text{IIII}}$ |	6	$\frac{6}{30}$	20%
TOTAL		30		100%

3. Using the completed Data Form, have students discuss the following questions: a. What is the best title for the Class Survey Math Sheet? b. Which dessert(s) did the class choose most often? c. Which dessert(s) were not chosen at all? d. Which dessert(s) received the highest percentages? e. Which dessert(s) should be purchased for the party?	3. Responses will be based on the collected data.
4. Using the steps involved in the development of the lesson, reinforce the concept that the way to find new information from given data is called <u>statistics</u>. Define statistics as the science of collecting, organizing and interpreting data.	
5. Elicit the basic steps involved in the collection and interpretation of data.	5. Step 1 - Collect data. Step 2 - Organize data in the form of a chart. Step 3 - Change information obtained from the data to a ratio and a percent. Step 4 - Using the final percentages, interpret the data to determine new facts.

<u>SUMMARY</u>

1. What do we call the science of collecting, organizing, and interpreting data?

2. Define: tally, mark, ratio.

SUGGESTED EXERCISES

Complete the table below and answer the questions.

TITLE: NUMBER OF MINUTES PER DAY STUDENTS IN MATH CLASS 217 LISTEN TO THE RADIO

TIME IN MINUTES	TALLY	NUMBER OR FREQUENCY	NUMBER / TOTAL SAMPLE	PERCENT
0-15	|			
16-30	||			
31-45	|||			
46-60	𝍸 |			
61-75	𝍸 𝍸			
76-90	||||			
91-105	||			
106-120				
121-135	|			
136-150				
151-165				
166-180				
TOTAL				

1. What percent of students listens to the radio 1 hour or more? 30 minutes? At least 150 minutes? 1/4 of an hour? 3/4 of an hour?

2. What percent of students watches television?

CLASS SURVEY DATA FORM

TITLE: DESSERT PREFERENCES OF MEMBERS OF A CLASS

FAVORITE DESSERTS	TALLY	NUMBER	NUMBER TOTAL CL. SIZE	PERCENT
BROWNIES				
CAKE				
CANDY				
COOKIES				
CUP CAKES				
FRESH FRUIT				
ICE CREAM				
ITALIAN ICES				
JELLO				
MELON				
PIE				
POTATO CHIPS				
PRETZELS				
PUDDING				
TOTAL				

UNIT IV

Lesson 2

Aim

To select a random sample

Motivation

Remove the 12 picture cards from a standard deck of playing cards. Using the remaining group of 40 cards (numbered 1 to 10, an ace being 1) ask the question:

If we pick a card from the deck, what number has the highest probability of being chosen? (Return the card after each pick.)

Materials

Standard deck of cards, 10 different colored beads, paper bag, spinner (divided into 8 equal sectors)

Development

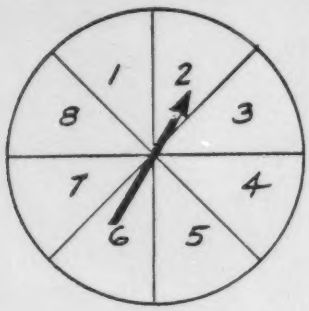
Teacher Activity	Student Activity
1. Using the problem in the motivation, elicit the need to collect data on an experimental basis.	
2. Discuss the reason why the deck of cards should be well shuffled.	
3. Have students draw one card at a time, record the outcome, return the card to the deck, re-shuffling the deck on a continuous basis. Inform the students that the card being drawn each time has been selected on a "random" basis.	3. Students should compose and fill out a frequency table.
a. After 10 trials, elicit from the class the percent of time each number has occurred.	
b. Would the same outcomes occur if 20 trials were made? 40 trials? Have students answer these questions by actually performing the experiments. In each experiment, have the students list the data in the form of a frequency table.	

c. Using the data collected in each card experiment, determine:

1. which cards occurred most often. least often.
2. if the number of trials affects the outcome.

4. Place 10 colored beads in a paper bag. Using the same procedure as in the experiment involving the cards, have students generate a frequency table using the bead selection as their criterion reference. Set up a table for experiments involving 10 trials, 20 trials, 40 trials.

5. Repeat the same type of experiment using the spinner below. Have students generate the frequency table based on 8 trials, 16 trials, 24 trials.



6. Using the experiments involving the cards, beads and spinner, elicit the definition of a <u>random sample</u>.

6. A random sample means that each member of the same population has an equal chance of being selected.

<u>SUMMARY</u>

Review how to determine a random sample.

SUGGESTED EXERCISES

FREQUENCY TABLE

TITLE: CHOOSING CARDS

ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN

1. Take a sheet of paper and cut it into ten equal parts. Number each of the parts, using the numbers one through ten. Fold each slip in half, and place them in a paper bag. Without looking, select a piece of paper. Record the number on a frequency table, and then return the paper to the bag. Use 30 trials.

   a. Which number had the highest frequency?

   b. Which number had the lowest frequency?

   c. If another 30 trials are made, what results would you expect? Explain.

2. When a thumbtack is tossed it will land either with the point up or with the point down.

   a. Predict which event is more likely.

   b. Toss a tack 25 times and record the results in a frequency table. Which event was the most frequent?

   c. How does your prediction compare with the experiment?

3. Two dice are tossed. The numbers showing are multiplied. The product is either even or odd.

   a. Predict the probability that the product is odd.

   b. Perform the experiment 20 times. Does your prediction hold?

Lesson 3

Aim

To use the mean (average) to interpret data

Motivation

A mechanic's salary for the past eight weeks was:
$192, $199, $210, $218, $187, $205, $217, $196.
What was his average weekly salary?

Development

Teacher Activity	Student Activity
1. a) Using the motivation problem, guide the students to find the arithmetic average, which is commonly called the <u>mean</u>, or the <u>arithmetic mean</u>.	1. a) $192 199 210 218 187 205 217 196 $1624 $\frac{1624}{8} = 203$ His average weekly salary was $203.
b) What does the mean or average salary tell the mechanic?	b) Possible responses: He earns about $203 per week. If the same number of dollars were given him in equal payments, he would receive $203 each week. He is earning the same as someone receiving $203 per week.
2. Other classroom activities can be arranged to have the students involved in organizing and interpreting data.	

For example:

Pupils' ages to the nearest birthday can be tabulated, and the average age of the group determined.

Pupils' heights can be measured, and the average height of the class computed.

If a scale is available, the averageweightof the group can be determined.

Summary

1. What is another name for the arithmetic average?
2. What is the arithmetic mean or average of 60 and 40?
3. How do we find the mean or average of a group of numbers?

SUGGESTED EXERCISES

1. Find the mean for each of the following sets of test scores:
   a. 60, 95, 70, 80, 65
   b. 0, 70, 75, 0, 85, 75, 80, 80, 75
   c. 60, 65, 70, 90, 100, 60, 80
2. In a walkathon food drive, a sophomore class collected the following contributions: $1.35, $.85, $.75, $1.65, $2.10, $1.55, $1.40, $.90, $1.90, $1.85, $2.00, $1.75, $.95, $1.20. Find the average contribution.
3. What is the average speed (m.p.h.) of an aircraft if it flies 2,500 miles in 6 1/4 hours?
4. Find the average number of letters in each word in this problem.
5. The average of three consecutive whole numbers is ten. What are the numbers?
6. Jane bowled 21 games, getting a total of 2562 pins. Rose bowled 15 games, getting 1785 pins. Who had the higher average? How much higher was it?

Lesson 4

Aim

To use the mode to interpret data

Motivation

One morning, a shoe store sold shoes in the following sizes:
7½, 7½, 8, 8, 9, 9, 6, 6, 5, 5, 6, 6½, 5½, 7½, 7½, 7, 5.
Which size was purchased most often?

Development

Teacher Activity	Student Activity
1. a) Guide the students to organize the data into a frequency table.	1. a)

Shoe Size	Frequency
5	3
5½	1
6	3
6½	1
7	1
7½	4
8	2
9	2

b) Which size shoe was purchased most frequently?

b) Size 7½ was purchased most frequently.

c) Explain that this measure of central tendency is called the mode. The mode is the number or score that appears most often.

c) The mode is 7½.

d) Have students realize that the most frequently sold size must be stocked in greater quantity.

2. a) Pose the question: Suppose the next customer purchased a size 6 shoe. Which size would then have been sold most often?

2. a) Size 6 and size 7½ would each have a frequency of four.

b) Explain that there can be more than one mode for a given set of data. If no item appears more often than the others. there is no mode.

b) Size 6 and size 7½ are each modes.

3. a) Using the information below, have students determine the mean and the mode.

ACE LUMBERYARD
SALARY SCALE

Salary	Number of Employees
$6,000	16
7,000	40
8,000	10
15,000	6
20,000	5
30,000	1
50,000	2

3. a) The mean salary is $9,700.

The mode is $7,000.

b) Which measure of central tendency - the mean or the mode - gives a more realistic basis for determining whether the salaries at the Ace Lumberyard are high enough to attract new employees?

b) The mode gives a more realistic interpretation of this data than the mean. The mean was affected by the high salaries and did not accurately represent the most common salaries ($6,000 - $8,000).

Summary

1. How do we determine the mode for a set of data?

2. What is the mode for this set of test scores?

50 50 65 80 100

EXTENDED EXERCISES

1. Have the students collect data involving their favorite sport, music group, color, etc. Using this data, have students determine the mode. Discuss how the mode might be useful in drawing conclusions.

2. Set up data in the form of a frequency table.
   Determine the most common profession of the Signers of the Declaration of Independence (MCDE).

Name	Occupation	Name	Occupation
John Adams	Lawyer	Samuel Adams	Politician
Josiah Bartlett	Physician	Carter Braxton	Planter
Charles Carroll	Planter	Samuel Chase	Lawyer
Abraham Clark	Lawyer - Farmer	George Clymer	Merchant
William Ellery	Lawyer	William Floyd	Farmer
Benjamin Franklin	Public Servant	Elbridge Gerry	Merchant
Button Gwinnett	Planter	Lyman Hall	Physician
John Hancock	Merchant	Benjamin Harrison	Planter
John Hart	Farmer	Joseph Hewes	Merchant
Thomas Heyward	Lawyer	Francis Hopkinson	Lawyer
Stephen Hopkins	Public Servant	Thomas Jefferson	Lawyer- Planter
Samuel Huntington	Lawyer	Francis Lightfoot Lee	Planter
Richard Henry Lee	Planter	Phillip Livingston	Merchant
Francis Lewis	Merchant	Thomas McKean	Lawyer
Thomas Lynch	Planter	Lewis Morris	Landowner
Arthur Middleton	Planter	John Morton	Surveyor
Robert Morris	Merchant	William Paca	Lawyer
Thomas Nelson	Planter	John Penn	Lawyer
Robert Treat Paine	Lawyer	George Read	Lawyer
Caesar Rodney	Landowner	George Ross	Lawyer
Benjamin Rush	Physician	Edward Rutledge	Lawyer
Roger Sherman	Merchant	James Smith	Lawyer - Iron Master
Richard Stockton	Lawyer	Thomas Stone	Lawyer - Planter
George Taylor	Iron Master	Matthew Thornton	Physician
George Walton	Lawyer	William Whipple	Merchant
William Williams	Pub.Servant-Merchant	James Wilson	Lawyer
John Witherspoon	Coll.Pres.-Clergyman	Oliver Wolcott	Lawyer
George Wythe	Lawyer		

Lesson 5

Aim

To use the median to interpret data

Motivation

A group of students took a test and received these marks:

90%, 65%, 0%, 100%, 70%, 0%, 80%, 95%, 85%.

Arrange these scores in order, from highest to lowest. Then find the mean and the mode. Do these measures give a good idea of how well the pupils did on the test?

Development

Teacher Activity	Student Activity
1. a) Discuss the importance of organizing data, either in a frequency table, or in order of size.	1. a) In size order: 100%, 95%, 90%, 85%, 80%, 70%, 65%, 0%, 0%.
b) What is the mode? How well does it reflect the results?	b) The mode is 0%. The mode does not reflect the results well because only two out of the nine students received less than 65%.
c) What is the mean? How well does it reflect the results?	c) The mean is 64%. The mean does not reflect the results well because nine of the nine students received a mark greater than 65%.
2. a) Inform students that, in a case such as this, the most appropriate measure of central tendency is the median. The median is the middle number of a set of scores arranged in ascending or descending order.	2.
b) Which will be the middle score in this list of nine scores?	b) The middle score is 80%. Four students have scores over 80%. Four students have scores less than 80%. The median is 80%.

3. a) Pose this problem: Find the median of this group of prices: 40¢, 50¢, 60¢, 40¢, 20¢, 70¢.

b) Guide students to see that, when the median lies between two numbers, we use the average (mean) of the two numbers.

3. a) 70¢
60¢
50¢
40¢ ← the middle
40¢
20¢

b) 40¢ + 50¢ = 90¢

$\frac{90¢}{2}$ = 45¢

The median price is 45¢.

Summary

1. To analyze a set of data, how should it be organized?
2. Which score is the median in a set of data?
3. How do we find the mean of two numbers?

SUGGESTED EXERCISES

1. MASONS' WORKSHEET:

	Number of Bricks per Hour		Number of Bricks per Hour
Mason A	20	Mason F	25
Mason B	10	Mason G	20
Mason C	30	Mason H	40
Mason D	40	Mason I	20
Mason E	20		

a) Arrange the data into a frequency table.

b) Find the mode, mean, and median.

c) Discuss the relative significance of these three different measures of central tendency.

2. For each of the following, set up a frequency table and find the mode, the mean, and the median.

2 (continued)

AMERICAN LEAGUE RUNS-BATTED-IN LEADERS

Year	Player and Club	RBI	Year	Player and Club	RBI
939	Ted Williams, Boston	145	1955	Rocky Bridges, Detroit	116
940	Hank Greenberg, Detroit	150		Jackie Jensen, Boston	116
941	Joe DiMaggio, New York	125	1956	Mickey Mantle, New York	130
942	Ted Williams, Boston	137	1957	Roy Sievers, Washington	114
943	Rudy York, Detroit	118	1958	Jackie Jensen, Boston	122
944	Vern Stephens, St. Louis	109	1959	Jackie Jensen, Boston	112
945	Nick Etten, New York	111	1960	Roger Maris, New York	112
946	Hank Greenberg, Detroit	127	1961	Roger Maris, New York	142
947	Ted Williams, Boston	114	1962	Harmon Killebrew, Minnesota	126
948	Joe DiMaggio, New York	155	1963	Dick Stuart, Boston	118
949	Ted Williams, Boston	159	1964	Brooks Robinson, Baltimore	118
	Vern Stephens, Boston	159	1965	Rocky Colavito, Cleveland	108
950	Vern Stephens, Boston	144	1966	Frank Robinson, Baltimore	122
	Walt Dropo, Boston	144	1967	Carl Yastrzemski, Boston	121
951	Gus Zernial, Chicago Philadelphia	129	1968	Ken Harrelson, Boston	109
952	Al Rosen, Cleveland	105	1969	Harmon Killebrew, Minnesota	140
953	Al Rosen, Cleveland	145	1970	Frank Howard, Washington	126
954	Larry Doby, Cleveland	126	1971	Harmon Killebrew, Minnesota	119

President and Political Party	Age at Inauguration
1. George Washington (F)	57
2. John Adams (F)	61
3. Thomas Jefferson (D-R)	57
4. James Madison (D-R)	57
5. James Monroe (D-R)	58
6. John Quincy Adams (D-R)	57
7. Andrew Jackson (D)	61
8. Martin Van Buren (D)	54
9. William H. Harrison (W)	68
10. John Tyler (W)	51
11. James Knox Polk (D)	49
12. Zachary Taylor (W)	64
13. Millard Fillmore (W)	50
14. Franklin Pierce (D)	48
15. James Buchanan (D)	65
16. Abraham Lincoln (R)	52
17. Andrew Johnson (D)	56
18. Ulysses Simpson Grant (R)	46
19. Rutherford B. Hayes (R)	54
20. James Abram Garfield (R)	49
21. Chester Alan Arthur (R)	50
22., 24. Grover Cleveland (D)	55
23. Benjamin Harrison (R)	55
25. William McKinley (R)	54
26. Theodore Roosevelt (R)	42
27. William Howard Taft (R)	51
28. Woodrow Wilson (D)	56
29. Warren G. Harding (R)	55
30. Calvin Coolidge (R)	51
31. Herbert Clark Hoover (R)	54
32. Franklin D. Roosevelt (D)	51
33. Harry S. Truman (D)	60
34. Dwight D. Eisenhower (R)	62
35. John F. Kennedy (D)	43
36. Lyndon Baines Johnson (D)	55
37. Richard M. Nixon (R)	56

3. A machine produced bolts with the following diameters, in inches: 0.67, 0.66, 0.67, 0.68, 0.65, 0.69, 0.66, 0.67, 0.68. Find the mean, median, and mode for this batch of bolts.

4. A 4-penny nail has a weight of 4 x 0.05 ounces, or 0.20 ounces. Find the mean, median, and mode of the weights of this group of 4-penny nails being tested as part of a quality control program: .020, .021, .019, .018, .021, .020, .020, .020, .019, .022, .020, .021, .020, .019.

5. The height, in inches, of the members of a basketball team, was reported as: 72, 70, 73, 71, 74, 73, 71, 73, 70, 76, 74, 72, 74, 74, 69. Find the mean, median, and mode.

6. Find the mean, median, and mode of this set of test scores: 50%, 80%, 60%, 40%, 80%, 65%, 50%, 65%, 30%, 80%. Which measure best reflects the work of the student? Which measure would the student probably want the teacher to use? Explain.

UNIT IV

Lesson 6

Aim

To define and use the concept of sample space

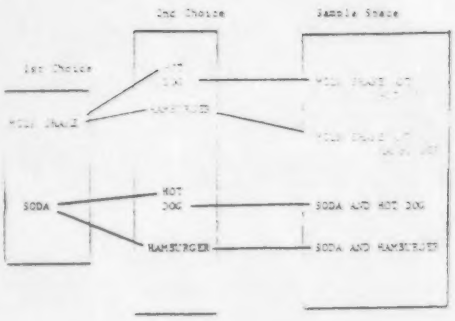
Motivation

The following is a copy of a luncheon menu. On the basis of selecting one item from each group, what is the number of ways of selecting lunch?

MENU

BEVERAGE:	MILK SHAKE SODA
SANDWICH:	HAMBURGER HOT DOG

Development

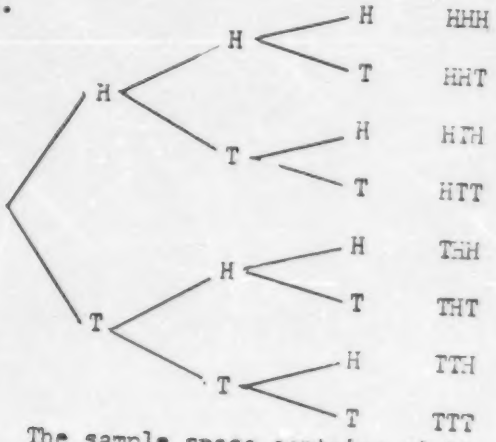
Teacher Activity	Student Activity
1. Using the problem in the motivation, elicit the concept of choice combinations.	
a. After writing all the possibilities to select a lunch, illustrate the systematic approach to listing the combinations in the form of "tree diagrams." Remind students that the list of all possible outcomes is called the sample space. Each outcome is called a simple outcome or simple event.	

b. Have the students list each simple event in the sample space.

b. Simple events:
1. Milkshake and hot dog
2. Milkshake and hamburger
3. Soda and hot dog
4. Soda and hamburger

2. a. Pose this problem:
Three coins are tossed.
Use a "tree diagram" to find all the possible outcomes.

2. a.



b. How many simple events are in the sample space?

b. The sample space contains eight simple events.

Summary

1. What do we call a list of all possible outcomes of an event?

2. What do we call each of the outcomes?

3. If you toss a coin once, how many events are in the sample space?

SUGGESTED EXERCISES

1. Arlene took a test with three true-false questions.
a) Make a tree diagram to show all the possible outcomes.
b) How many possible outcomes are there?
c) How many of these outcomes would give three correct answers?

2. At lunch, employees had a choice of three kinds of meat (ham, veal, or pork) and two vegetables (peas or potatoes).
a) Make a tree diagram to show all possible combinations.
b) How many of the combinations have no veal or peas (careful)?

3. Nora had a spinner with equal spaces for the numbers from one to six. Each spin gave either an odd or even number. List the sample space for all the possible outcomes of odd or even numbers in three spins.

Lesson 7

Aim

To develop the concept of probability

Motivation

If a six-faced die is tossed, what is the probability of rolling a three?

Materials

A die, colored pencils

Development

Teacher Activity	Student Activity
1. Discuss the motivation.	1.
a) How many possible numerals can appear.	a) Any of six numerals might appear: 1, 2, 3, 4, 5, or 6.
b) Which numeral is most likely to appear?	b) Each of the six numerals is equally likely.
c) Out of the six possible results, how many are "threes"?	c) One out of the six is a "three".
d) How would you describe the probability of rolling a three?	d) There is one chance out of six.
e) How can we express this probability as a ratio or fraction?	e) A three might appear 1/6 of the time.
f) Inform students that the probability is equal to the fraction: $\frac{\text{number of successful outcomes}}{\text{number of possible outcomes}}$. The number of possible outcomes is equivalent to the number of simple events in the sample space.	f) Probability of a three: $P(3) = 1/6$

2. a) Pose the problem: Suppose four red and eight blue pencils are placed in a case. If one pencil is chosen at random, what is the probability of selecting a red pencil? a blue pencil?

2.

b) How many pencils can be chosen?

b) Any of 12 pencils can be chosen.

c) How many of these are red?

c) Four out of the 12 pencils are red.

d) What is the probability of selecting a red pencil?

d) Probability (red) = $\frac{4}{12} = \frac{1}{3}$

e) What is the probability of selecting a blue pencil?

e) Probability (blue) = $\frac{8}{12} = \frac{2}{3}$

3. Inform students that the symbol P is often used to represent "Probability".

3. Probability (blue) = P (blue)

4. Pose the problem: Mr. Wallace has a $1 bill, a $5 bill, and a $10 bill in his pocket. He reaches into his pocket and takes one out at random. What is the probability he:

4.

a) selects a $1 bill?

a) P ($1) = $\frac{1}{3}$

b) selects a $5 bill?

b) P ($5) = $\frac{1}{3}$

c) selects a $10 bill?

c) P ($10) = $\frac{1}{3}$

d) selects a bill worth less than $15?

d) P (less than $15) = $\frac{3}{3} = 1$

e) selects a bill worth more than $15?

e) P (more than $15) = $\frac{0}{3} = 0$

5. Have the students study and discuss these results. Guide them to conclude that:

a) The probability of an event that is certain to occur is 1. — a) $\frac{3}{3} = 1$

b) The probability of an event that is impossible is 0. — b) $\frac{0}{3} = 0$

c) The sum of all the probabilities is 1. — c) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

Summary:

1. What do we mean by a probability of 1/2 ?
2. What do we mean by a probability of one?
3. What do we mean by a probability of zero?
4. When tossing a coin, what is the probability of a tail?

SUGGESTED EXERCISES

1. Three slips of paper are placed in a bag, each the same size and shape. On one there is an 'A', on another a 'B', on the third, a 'C'. One of these slips is chosen, placed back in the bag, and another chosen. Make a tree diagram and list all possible outcomes.
   a. How many possible outcomes are there?
   b. How many of the outcomes are favorable for two A's?
   c. What is the probability of picking two A's?
   d. How many of the outcomes are favorable for an A on the first pick and then a B on the second pick?
   e. What is the probability of picking first an A and then a B?
   f. How many outcomes are favorable to B being selected on the second pick?
   g. What is the probability of B being picked on the second pick?
   h. What is the probability of getting at least one A in two picks?
   i. What is the probability of getting at least one B?
   j. What is the probability of getting no C's in the two picks?
   k. What is the probability of getting a C on the second pick?
   l. What is the probability of getting one A and one C, in any order?

2. There are two blouses hanging in your closet, a red one and a blue one. It is dark in the closet, and you take one without seeing the color. What is the probability that you picked a blue blouse?

3. Chuck has eight blue marbles and two red marbles in a bag. If he takes one out without looking, what is the probability he will select a:
a) blue marble?
b) red marble?
c) green marble?

4. A deck of 52 playing cards is shuffled. The top card is turned over. What is the probability it is:
a) a black card?
b) a heart?
c) a king?
d) a queen of spades?

5. A die is rolled. What is the probability that the number of dots showing is:
a) five?
b) an odd number?
c) greater than zero?
d) greater than six?

6. After a quality control check, it was determined that 96% of the parts produced by a particular machine were acceptable. If this machine produces 2000 parts each day, how many will probably be acceptable? What is the probability that a part will be acceptable?

Lesson 8

Aim

To determine probabilities through experiments and by using addition

Motivation

Flip a coin ten times. Write down the number of heads and the number of tails you get.

Materials

Coins, cards numbered 1 through 10, dice

Development

Teacher Activity	Student Activity
1. Have the results obtained by the individual students tallied on the chalkboard. After each group of five students, have the number of heads and number of tails calculated.	1. Typical results:

	HEADS	TAILS
Bob	6	4
Jean	5	5
Sue	3	7

2. Consider questions such as these:	2.
a) What is the probability of a head for a single toss?	a) $\frac{1}{2}$
b) How many heads would we expect in ten tosses?	b) $\frac{1}{2} = \frac{5}{10}$ We would expect five heads.
c) Do the individual results always match the expected results?	c) No. Many of the results are different from the predicted results.
d) As more and more trials are made, do the results get closer to, or futher from, the predicted result?	d) As the total number of trials increases, the results get closer to 1/2.

3.	Have the students use ten cards numbered from one through ten to conduct a similar experiment. Have students determine the experimental probability of selecting a card with the number six. Determine P(6) for ten trials; 20 trials; 30 trials; etc.	3. $P(6) = \frac{1}{10}$
4.	Discuss the probability of drawing <u>either</u> a 6 <u>or</u> a 4. Elicit that P(6 or 4) will equal: P(6) + P(4).	4. $P(6) = \frac{1}{10}$ or 0.1 $P(4) = \frac{1}{10}$ or 0.1 P(6 or 4) = 0.1 + 0.1 = 0.2
5.	Have students generalize the result. Inform them that events such as these are called <u>mutually exclusive</u>. Mutually exclusive events are such that when one happens the other(s) cannot happen.	5. P(A or B) = P(A) + P(B)

<u>Summary</u>

1. Do the results of an experiment with random choices always match our predictions? Explain.
2. What happens to the results of an experiment as the number of trials increases?
3. How can we compute the probability of two mutually exclusive events if either can occur?

SUGGESTED [illegible]

1. Roll a die a dozen times, and record the result of each roll. Then roll it another dozen times, again recording the results. Do this five times. Compare the actual results with the expected results. Then add all your results. Now compare these totals with the predicted results.

2. A set of cards is numbered one through eight. If a card is selected randomly, what is the probability of picking:
a) a seven?
b) a five or six?
c) a one, two, or four?
d) an even number?
e) a number greater than four?
f) a number less than four?
g) a number less than nine?
h) a number greater than nine?

3. In his pocket, John has six dimes that were minted in the following years: 1973 1973 1975 1978 1979 1980. What is the probability that a dime selected randomly is dated:
a) 1978?
b) 1973?
c) 1979 or 1980?
d) 1973 or 1975?
e) 1976?
f) between 1972 and 1977 ,exclusively?
g) before 1974?
h) before 1975?
i) after 1975?
j) after 1965?

4. If we use a deck of 52 playing cards, what is the probability of randomly drawing:
a) an ace?
b) a king?
c) an ace or a king?
d) a heart?
e) a spade?
f) a heart or a spade?
g) a red card?
h) a red or black card?
i) a jack, queen, or king?
j) anything but an ace?

Lesson 9

Aim

To compute probabilities involving multiplication

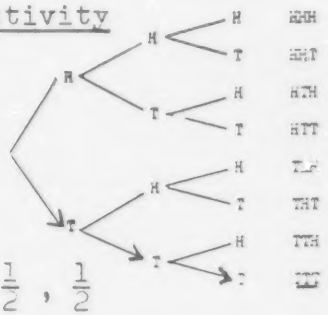
Motivation

A coin is tossed three times. Use a tree diagram to determine the probability that all three tosses result in tails? How can we compute this probability?

Materials

Overhead transparencies or prepared drawings of series and parallel circuits.

Development

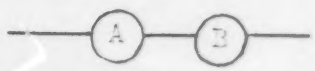
Teacher Activity	Student Activity
1. Elicit that the probability of three tails is 1/8.	
2. a) On the first toss, what is the probability of a tail? On the second toss? On the third toss?	2. a) $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$
b) How could we use these three probabilities to determine the probability of three tails?	b) By multiplying, because: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
c) Inform students that events such as these are called <u>independent events</u>. An independent event is one whose outcome does not affect the outcome of the other event(s).	c) The second and third tosses of the coin are independent of the results of the first toss.
d) The probability of independent events occurring is equal to the product of the probabilities of the individual events.	d) P(A and B) = P(A) x P(B)

3. a) Inform students that the reliability of electrical circuits is often found by multiplying probabilities.

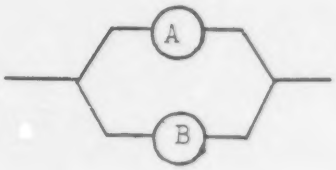
b) Present students with diagrams of a series circuit and a parallel circuit. Explain that a series system works only when all its components work, while a parallel system works when one or more of its components work. If component B were to fail in the series circuit, the circuit would fail. But if component B were to fail in the parallel circuit, the circuit would work.

3.

SERIES CIRCUIT:



PARALLEL CIRCUIT:



4. Have the students draw a table to show the various possible results for a pair of two component systems.

4.

SAMPLE SPACE	SERIES CIRCUIT	PARALLEL CIRCUIT
A WORKS and B WORKS	WORKS	WORKS
A WORKS and B FAILS	FAILS	WORKS
A FAILS and B WORKS	FAILS	WORKS
A FAILS and B FAILS	FAILS	FAILS

5. a) If the probability that component A works is .9, what is the probability that it fails? Why?

5. a) P(A WORKS) = .9

P(A FAILS) = .1

b) If the probability that component B works is .8, what is the probability that it fails? Why?

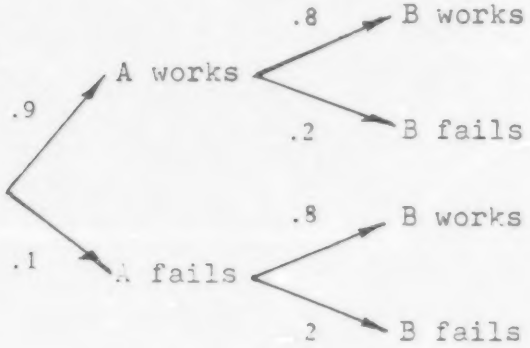
b) P(B WORKS) = .8

P(B FAILS) = .2

6. a) Inform students that the reliability of a series circuit and a parallel circuit using components A and B can be determined by using the probability rates.

b) Have the students recall that the probability of independent events can be determined by using multiplication.

c) Use a tree diagram to help the students visualize the various probabilities.



P(A works and B works) = .9 x .8 = .72

P(A works and B fails) = .9 x .2 = .18

P(A fails and B works) = .1 x .8 = .08

P(A fails and B fails) = .1 x .2 = .02

7. a) For which of these outcomes does the <u>series</u> circuit work?

b) Find the probability that the series circuit works.

c) For which of these outcomes does the <u>parallel</u> circuit work?

d) Find the probability that the parallel circuit works.

7. a) The series circuit works only for the first outcome listed.

b) P(series works) = .72

c) The parallel circuit works for the first three outcomes listed.

d) P(parallel circuit works) = .72 + .18 + .08 = .98

e) Which type of circuit has the greater reliability?

e) The parallel circuit is more reliable since it functions 98% of the time.

Summary

1. How can we find the probability of independent events?

SUGGESTED EXERCISES

1. What is the probability of rolling a six on a die (a) twice in a row? (b) three times in a row?

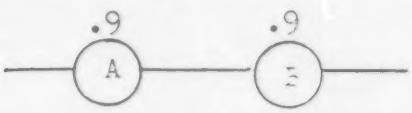
2. What is the probability of tossing a coin and getting heads (a) three times in a row? (b) five times in a row?

3. A spinner has five equal spaces numbered 1 through 5. What is the probability of spinning a 4 three consecutive times?

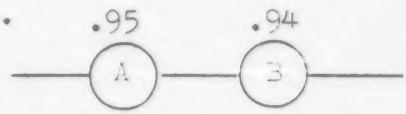
4. Three ordinary dice are rolled. What is the probability:
   a) all three are 2's?
   b) all three are even numbers?
   c) all three are greater than four?

5. Find the probability that each of these circuits works.

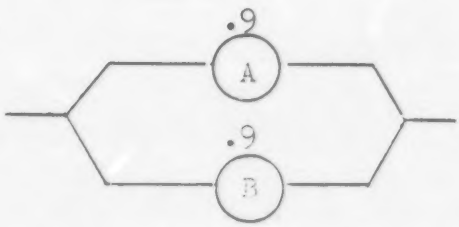
a.



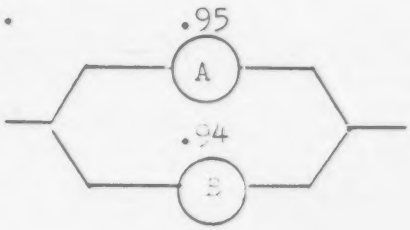
b.



c.



d.



6. If two dice are rolled, what is the probability of getting two fives?

Unit V

PERIMETERS, AREAS, AND VOLUMES OF GEOMETRIC FIGURES

Lesson 1 To review and reinforce the concept of perimeter (GHSM p. 39)*

Lesson 2 To calculate the circumference of a circle (GHSM p. 42)*

Lesson 3 To calculate the area of a rectangle using the counting method (GHSM pp. 39-42)*

Lesson 4 To calculate the area of a rectangle using the formula A = lw (GHSM pp. 41-42)*

Lesson 5 To calculate the area of a parallelogram and a triangle (GHSM pp. 40-41)*

Lesson 6 To calculate the area of a circle (GHSM p. 42)

Lesson 7 To learn the concept of volume (GHSM pp. 65-66)*

Lesson 8 To calculate the volume of a rectangular solid using the formula V = lwh (GHSM pp. 65-66)*

Lesson 9 To calculate the volume of a circular cylinder (GHSM pp. 65-66)*

Lesson 10 To calculate the volume of a triangular prism (GHSM pp. 65-66)*

Lesson 11 To calculate the surface area of a solid

Lesson 12 To introduce the units used in liquid measurement

*General High School Mathematics, New York State Department of Education, August 1970.

UNIT V

PERIMETERS, AREAS AND VOLUMES OF GEOMETRIC FIGURES

Lesson 1

Aim

To review and reinforce the concept of perimeter

Motivation

A builder is planning to buy galvanized fencing for a rectangular plot of ground. What does he need to know about the plot to decide how much fencing he must buy?

Development

Teacher Activity	Student Activity
1. From the problem in the motivation, elicit the need to calculate the total distance around the plot. Define the distance around an enclosed figure as the perimeter.	
2. Using a prepared acetate containing several geometric shapes with their dimensions, have the students determine the perimeters.	
Geometric figures should include commonly used polygons such as triangles, rectangles, and quadrilaterals.	
3. Use suggested exercises 1 and 2.	

SUMMARY

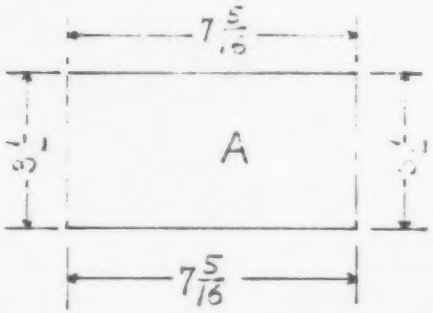
1. Review definition of perimeter.

2. Review how to calculate the perimeter of commonly used polygons.

(NOTE: All the measurements on the following pages are in inches.)

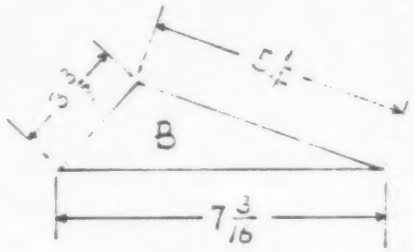
SUGGESTED EXERCISES

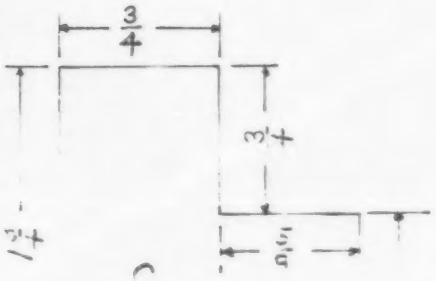
1. Calculate the perimeter of each of the figures below. Place your answer next to the proper letter listed down the center of the sheet.



A_____

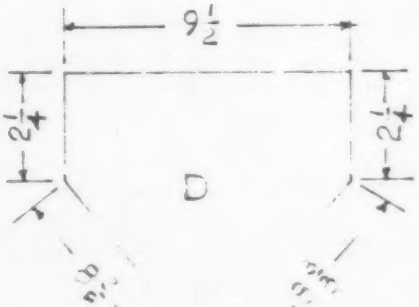
B_____

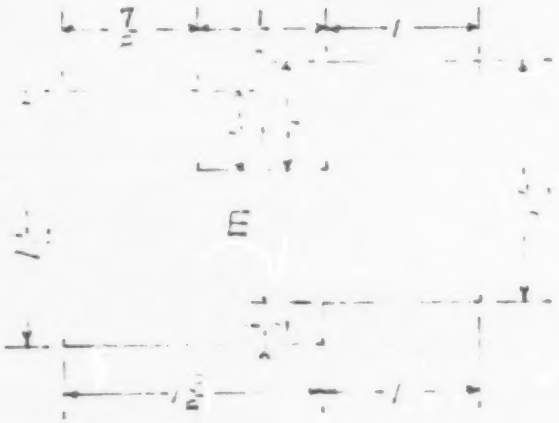




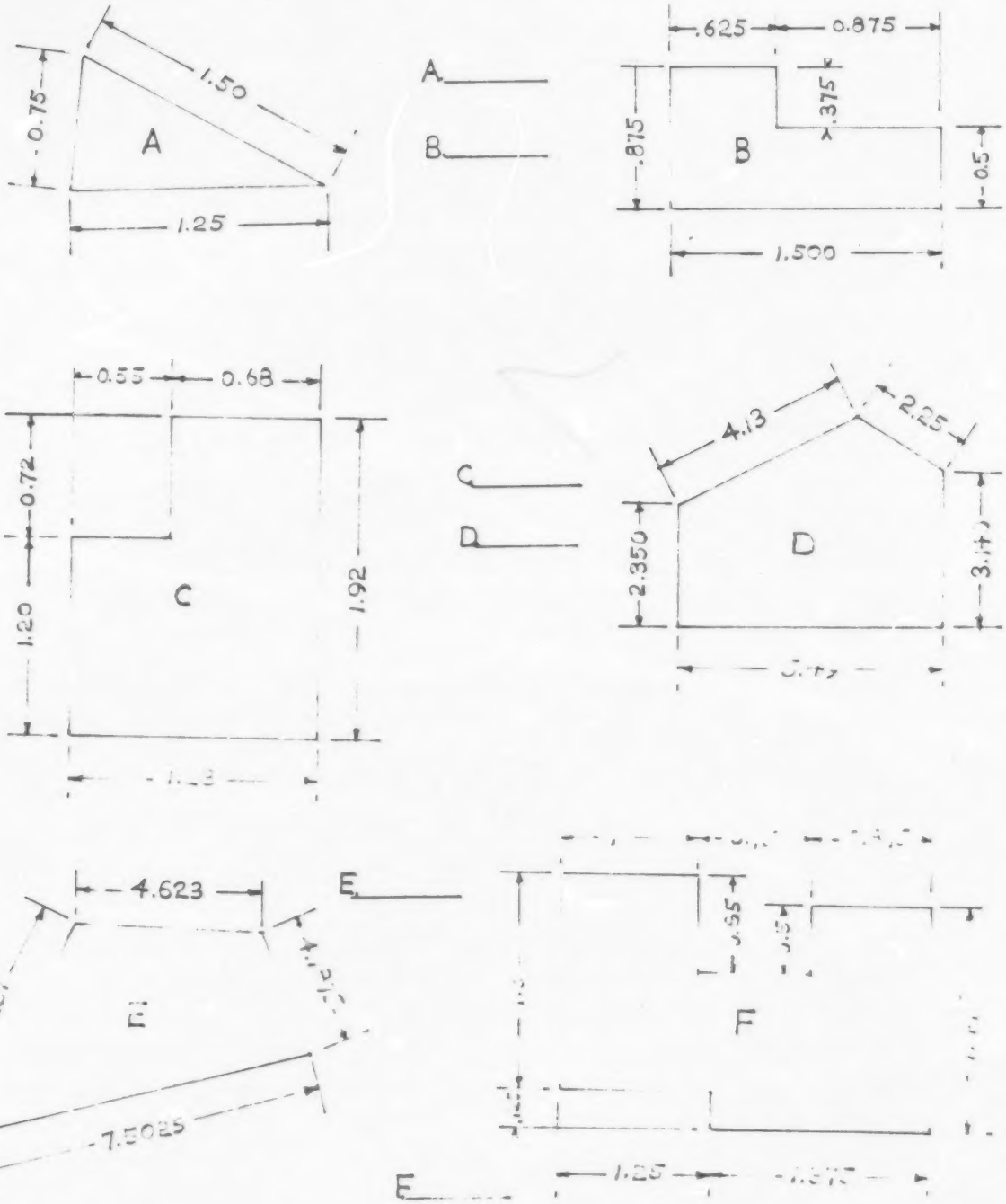
C_____

D_____

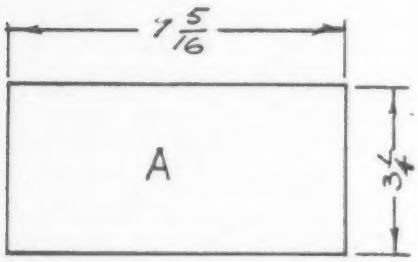




2. Calculate the perimeter of each of the figures below. Place your answers next to the proper letter listed down the center of the sheet.

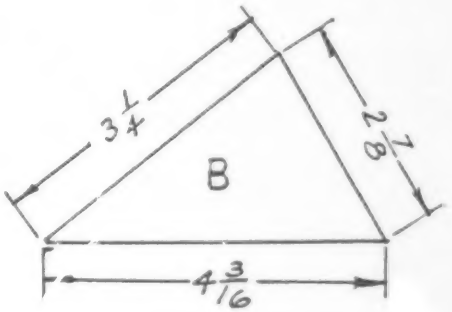


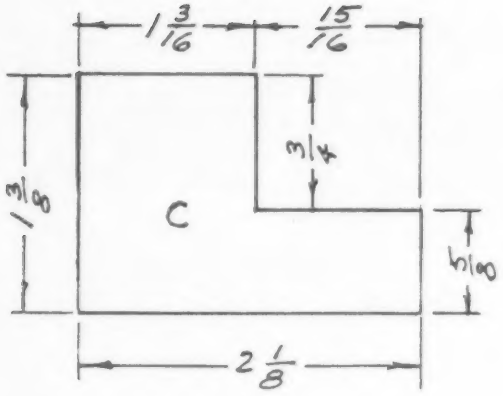
3. Calculate the perimeter of each of the figures below. Place your answers next to the proper letter listed down the center of the sheet.



A _____

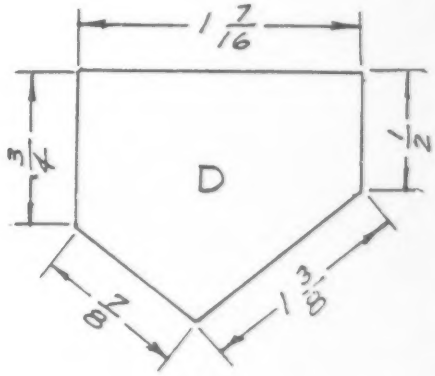
B _____

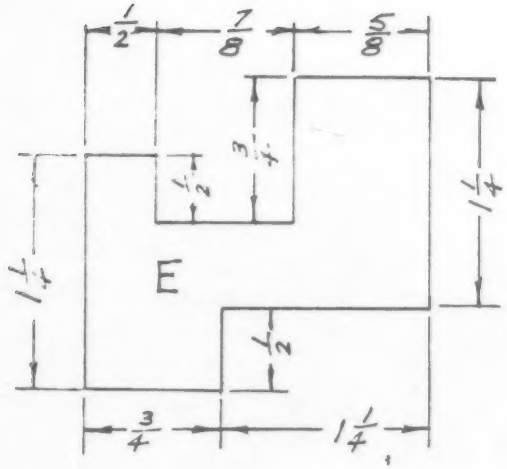




C _____

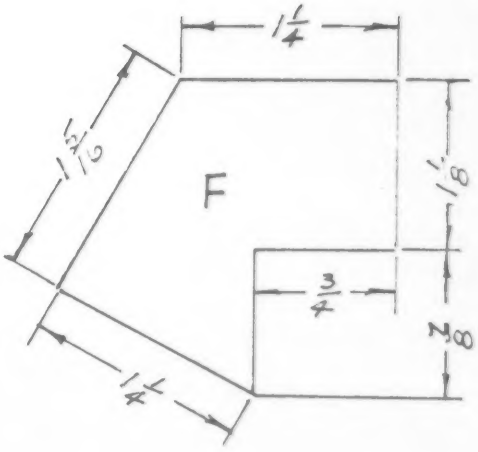
D _____



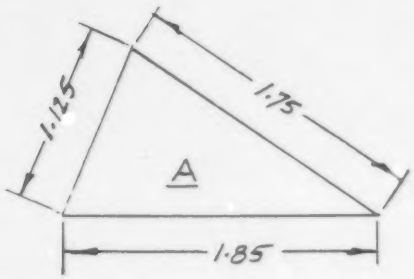


E _____

F _____

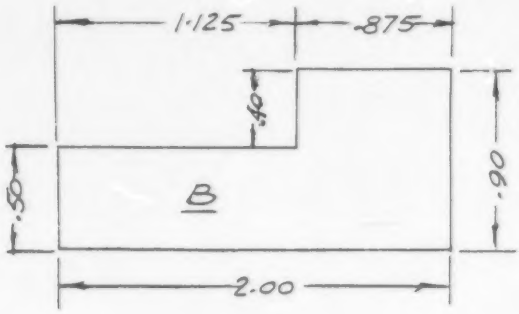


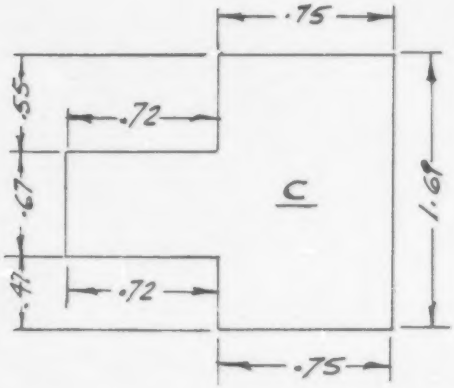
4. Calculate the perimeter of each of the figures below. Place your answers next to the proper letter listed down the center of the sheet.



A _____

B _____





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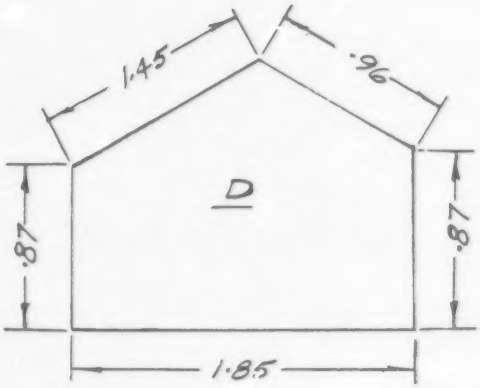
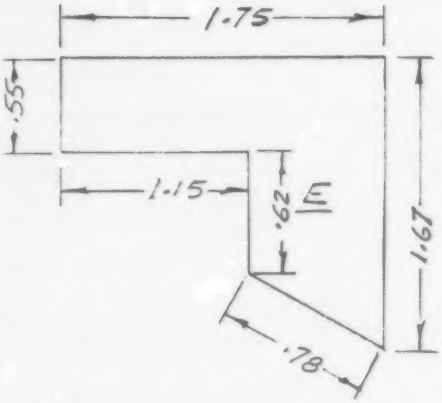
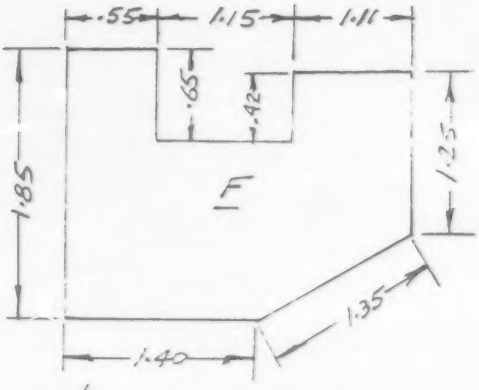




E _____

F _____



G _____

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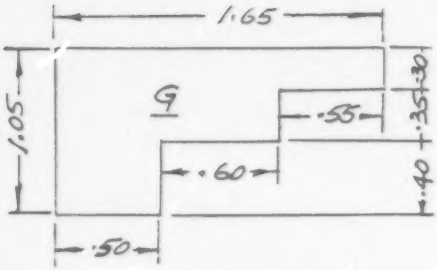
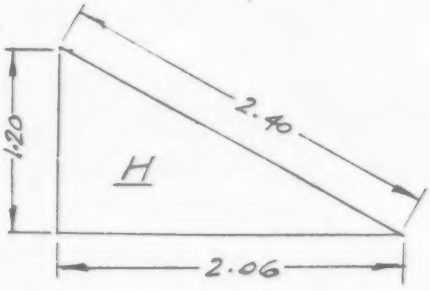




Lesson 2

Aim

To calculate the circumference of a circle

Motivation

Draw a rectangle, triangle, and circle on the chalkboard, using simple dimensions. Challenge the students to find the perimeter of each figure.

Development

Teacher Activity	Student Activity
1. Elicit that the distance around the circle cannot be found as readily as the perimeter of a polygon.	1. Students should give the perimeter of the rectangle and the triangle.
2. Have the students recall the meaning of the terms: radius, diameter, chord, and circumference.	2. The radius is a line segment from the center of the circle to any point on the circle. The diameter is a chord which passes through the center of the circle. The circumference is the distance around (perimeter) the circle.
3. Provide several models of circles (wheels, pizzas, cans, templates, etc.). Using a tape measure, a string, or a yardstick, have the students measure each circumference and diameter.	3. Students record results:
4. Have the students compare the circumference and diameter of each object measured. Working in small groups, or using a calculator, have the students calculate the ratio C/D to two decimal places.	4. The circumference is always a little more than three times as large as the diameter. $\frac{C}{D}$ is slightly over 3.1 for each circle measured.

			$\frac{C}{D}$ or C ÷ D
Record	37 5/8"	12"	
Cup	9 1/2"	3"	

5. Inform the students that this ratio is called π (pi). We normally use the approximations:
$\pi = 3.14$ or $\frac{22}{7}$

5. $\frac{C}{D} = \pi$

$\pi = 3.14159265358979323846264...$

6. Since the circumference is π times the diameter, we can express this as $C = \pi d$.

7. Because, in any circle, $d = 2r$, we can also use the formula $C = 2\pi r$.

8. Pose these problems:
a) What is the circumference of a circle whose diameter is 20 inches ($\pi = 3.14$)

b) What is the circumference of a circle whose radius is 14 inches
($\pi = 22/7$)

8. a) $C = \pi d$

$C = 3.14 \times 20$

$C = 62.8$

Circumference is approximately 62.8 inches.

b) $C = 2\pi r$

$C = 2 \times \frac{22}{7} \times 14$

$C = 88$

Circumference is about 88 inches.

Summary

1. What is the radius of a circle?
2. What is the diameter of a circle?
3. What do we call the perimeter of a circle?
4. What is the formula for the circumference of a circle?
5. What values do we use to approximate π?

SUGGESTED EXERCISES

1. Calculate the circumference of each of the following circles:

   a. Diameter = 21 inches

   b. Diameter = 30 feet

   c. Diameter = 5 meters

   d. Diameter = 14 centimeters

   e. Radius = 7 yards

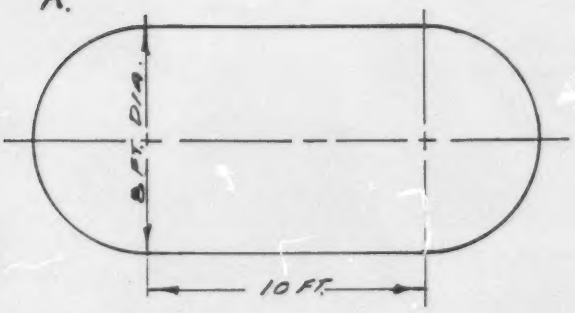
   f. Radius = 5 meters

   g. Radius = 4 1/2 inches

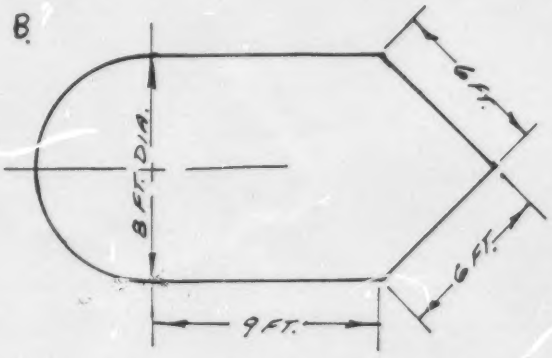
   h. Radius = 6.5 meters

2. Find the perimeter or circumference for each of the following figures:

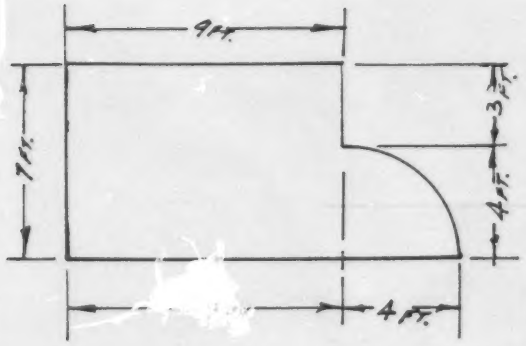
A.



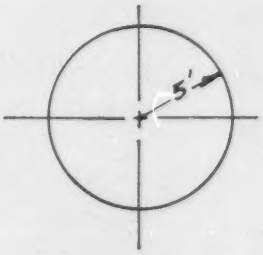
B.



C.



D.



Lesson 3

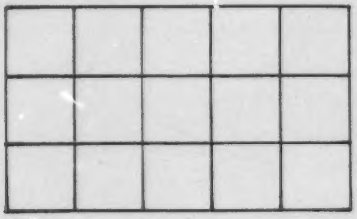
Aim

To develop the concept of area through counting the number of square units

Motivation

A.

B.



Two strips of metal, pictured above, are to be plated with gold. Strip A is 3 inches by 6 inches, and strip B is 2 inches by 7 inches. If the plating is to be of the same thickness on each strip, which will require the most gold?

Laterials

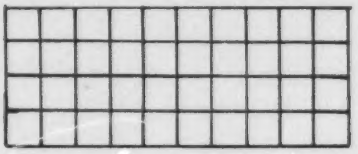
Models of square inches, square feet, square yard, square centimeters.

Development

Teacher Activity	Student Activity
1. Elicit that the strip with the greater surface will require the most gold. Since each of the squares pictured is the same size, the total amount of surface can be measured by counting the number of squares each strip contains.	1. Strip A will require more gold. Strip A has "18 squares" of surface, while strip B has only "14 squares" of surface.
2. Inform students that the total number of unit squares in a figure is called its area.	

2. a) Pose the problem: A hallway is 10 feet long and 4 feet wide. Draw a diagram to show this and determine the area of the hallway.

2. a)



b) Have the students realize that area can be measured in square feet, square yards, square centimeters, square meters, etc. The unit we choose is generally based upon the dimensions of the figure being measured.

b) The hallway has an area of 40 square feet.

3. To reinforce the distinction between "square units" and the shape of a figure, show students models such as these, drawn to correct scale, in square inches.

3.

a) Area is five square inches.

b) Area is four square inches.

a)

b)

4. a) Using models of square inches, square feet, and square yard, have students develop a list of the relationships between the most common units of area.

4. a) 1 sq ft = 144 sq in

1 sq yd = 9 sq ft

1 sq m = 10,000 sq cm

b) Introduce the use of the symbols such as "ft^2" to represent "square feet".

b) $1\ ft^2 = 144\ in^2$

$1\ yd^2 = 9\ ft^2$

$1\ m^2 = 10{,}000\ cm^2$

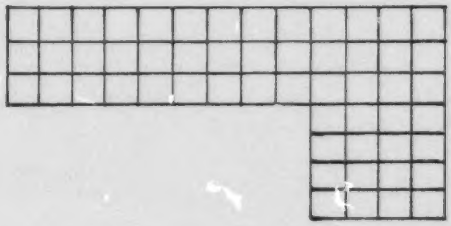
Summary

1. What kind of units are used to measure area?

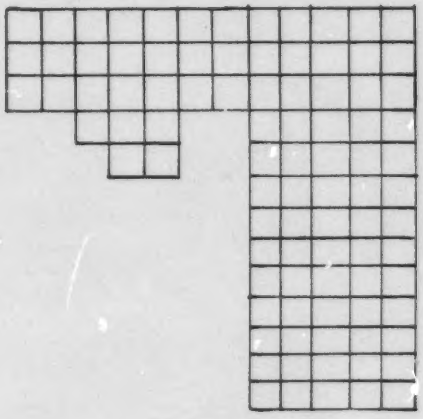
2. How many square feet are there in one square yard?

SUGGESTED EXERCISES

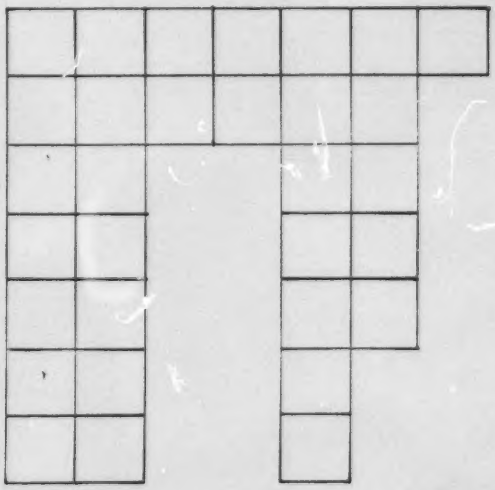
1. a. If the length of the side of each box is 1 inch, how many square units does each box represent?_____
   b. What is the unit of area in this example?_____
   c. How many boxes are there in the figure? _____
   d. What is the area of the figure? _____
   e. Why? _____



2. a. If the length of the side of each box is 1 foot, how many square units in each box? _____
   b. What is the unit of area in this example? _____
   c. How many boxes are there in the figure? _____
   d. What is the area of the figure?_____
   e. Why?_____



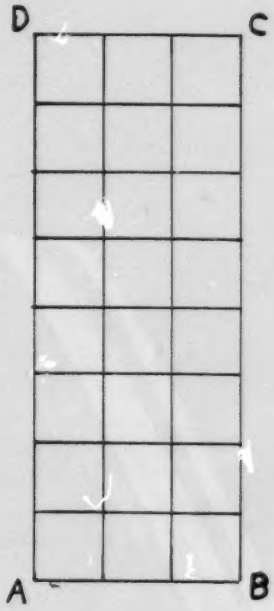
3. a. If the length of the side of each box is 1 centimeter, how many square units in each box? _____
   b. What is the unit of area in this example? _____
   c. How many boxes are there in the figure? _____
   d. What is the area of the figure? _____
   e. Why? _____



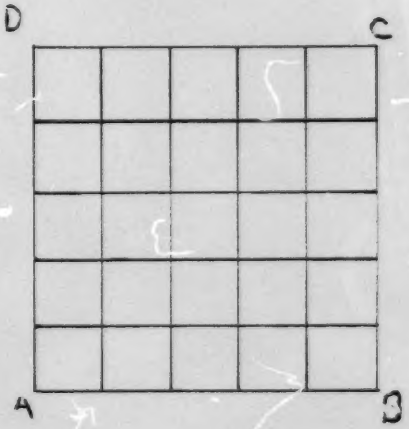
4. a. How many boxes are there in rectangle ABCD? _____
   b. If the side of each box is 1 inch long, what is the area of ABCD? _____
   c. How long is side AD? _____
   d. How long is side DC? _____
   e. What name do we give to the length of side AD? _____
   f. What name do we give to the length of side CD? _____
   g. Multiply: BASE times HEIGHT _____
   h. Which answer is this the same as? _____
   i. The easy way to find the area of a rectangle is to multiply

   _____

   (two answers: A= lw or A= bh)



[illegible]
[illegible] _____
b. If the side of each small square measures 1 inch, what is the area of ABCD? _____
[illegible] long is DC? _____
[illegible] long is AD?
[illegible] will tell you [illegible] about the length and the width of a square? _____
f. How many sides do you have to know to find the area of a square? _____



Lesson 4

Aim

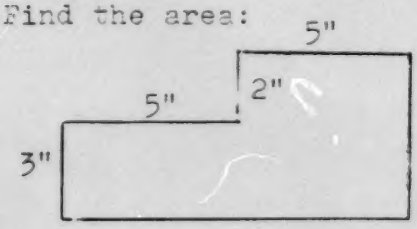
To calculate the areas of rectangles and squares using the formulas:
$A = lw$ and $A = s^2$

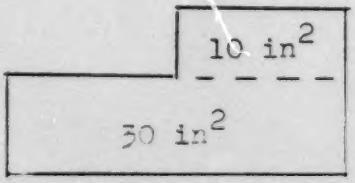
Motivation

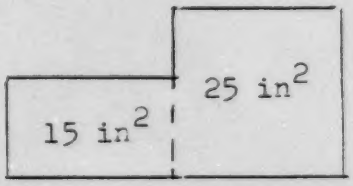
The roof of a small flat shed is four yards long and three yards wide. How many square yards of tar paper will be required to cover the entire roof?

Development

Teacher Activity	Student Activity
1. Based upon their work with the Suggested Exercises for Lesson 3, many pupils will have discovered the pattern for determining the area of a rectangle.	1. Area of a rectangle = length x width, or, base x height.
2. Have the students express the rule they have discovered as a formula.	2. For a rectangle: $A = lw$ $A = bh$
3. Guide the students to use the proper procedures for evaluating a formula. a. Write the formula. b. Substitute the values. c. Do arithmetic. d. Express the answer.	3. For the motivation problem: a. $A = lw$ b. $A = 4 \times 3$ c. $A = 12$ d. We need 12 sq yd of paper.
4. Present the students with an irregular figure composed of rectangles. Have them demonstrate different approaches. Find the area:	







Total area is 40 in^2.

5. a) Have the students find the area of a square, 6 cm by 6 cm.

5. a) $A = lw$
$A = 6 \times 6$
$A = 36$
The area is 36 cm^2.

b) Guide the students to realize that, since the length and width of a square have the same dimensions, only one side has to be measured. The formula for area of a square can therefore be written as $A = s \times s$ or $A = s^2$

b) $A = s^2$
$A = 6^2$
$A = 36$
The area is 36 cm^2.

Summary

1. What is the formula for area of a rectangle?
2. What is the area of a rectangle 5 feet by 2 feet?
3. What is the area of a square, each of whose sides is 10 meters?

SUGGESTED EXERCISES

1. The playground shown below is to be paved. The cost of paving is as follows:

Play area $3.50 per sq. ft.
Fencing area $4.25 per sq. ft.
Indoor area $5.50 per sq. ft.

Find the cost of paving each area. Find the total cost of paving the playground.

2. Find the area of the rectangles whose dimensions are listed below:

   a. 14 meters by 16 meters

   b. 142 centimeters by 60 centimeters

   c. 12 feet by 15 feet

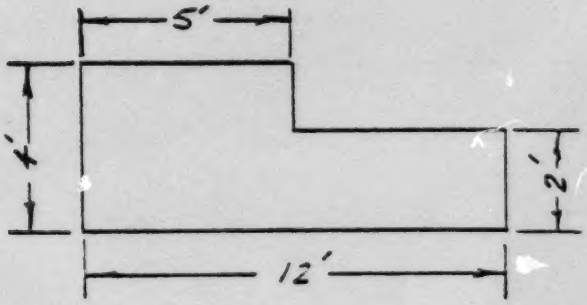
   d. 9 yards by 16 yards

   e. 9 feet 6 inches by 12 feet 4 inches

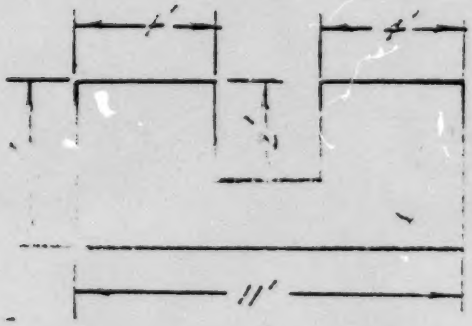
   f. 2-1/2 miles by 1-1/2 miles

3. Find the area of each of the following figures:

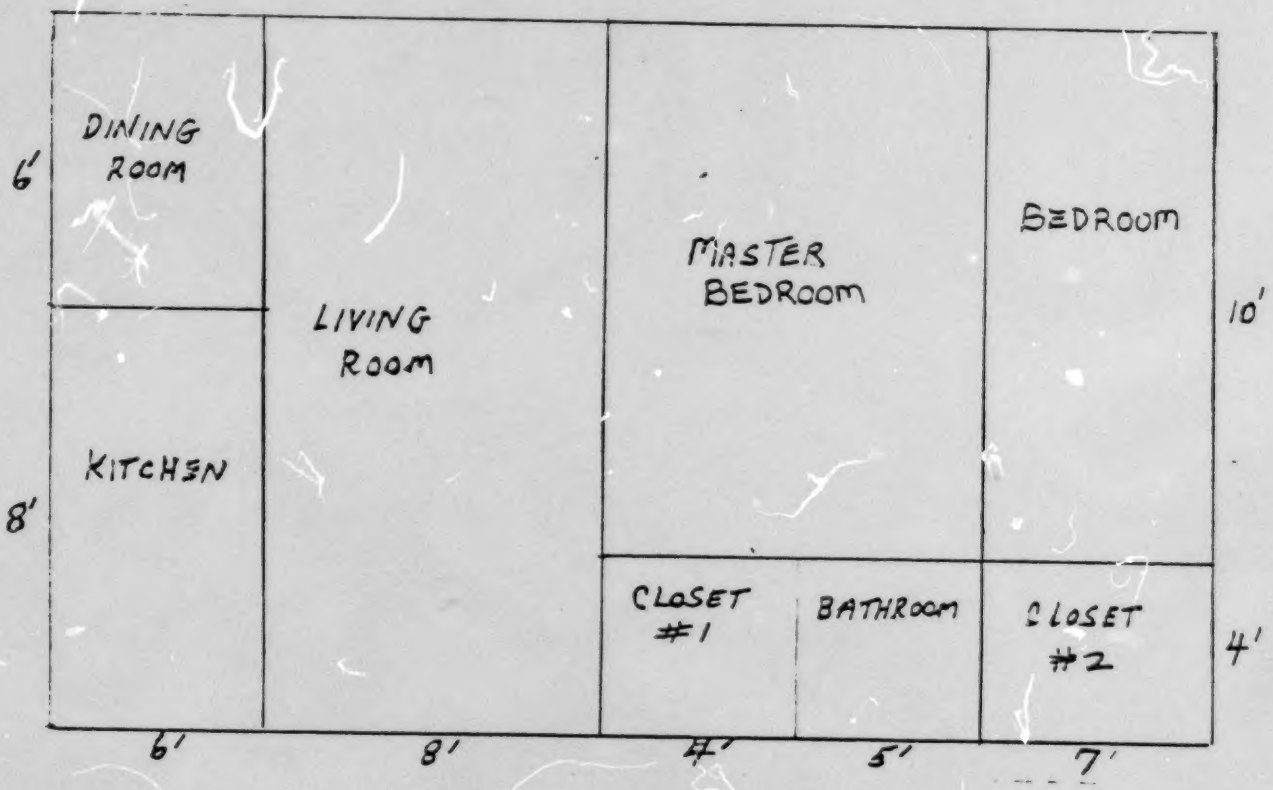
   a.



   b.



4. The following is a floor plan of an apartment.



a. Find the length and width of the living room.
b. Find the area of the living room.
c. Find the cost of carpeting the living room, if the carpet sells for $2 per square foot.
d. Find the area of closet #2.
e. Find the area of the bedroom.
f. Find the cost of carpeting the bedroom if the carpet sells for $1.75 per square foot.
g. Which part of the apartment has dimensions which show that it is a square?
h. Find the area of the kitchen in square inches.
i. If the cost of a 9" x 9" floor tile is 75¢, find the cost of covering the kitchen floor.

5. How many square yards of carpet must be ordered to cover a 12' x 15' room?

6. A rectangle whose area is 375 square feet has a length of 30 feet. Find its width.

7. A square has an area of 400 square inches. Find the perimeter of the square.

8. You wish to carpet your living room floor. It is rectangular in shape, 6 m x 5 m.
What is its area? _____

9. A woman wishes to cover the top of a table with plastic. What is the area to be covered if the table top measures 1 meter by 4 meters?
_____

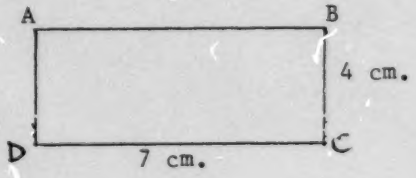
10. A man has a can of ceiling paint which will only cover a ceiling whose area is not larger than 150 sq. meters. Will he have enough paint to cover a ceiling whose length is 15 meters and whose width is 9 meters? _____

11. Anna is going to use 1-cm tiles to completely cover the square top of a lamp table. The table measures 30 cm. on each side. How many tiles will be needed to cover the table top? _____

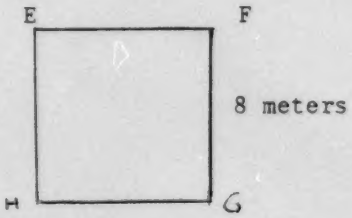
12. The bottom of a swimming pool is going to be painted with blue paint. What is its area if it is 20 meters long and 18 meters wide? _____

13. How many 2-meter square tiles are needed to cover an 8-meter square floor? _____

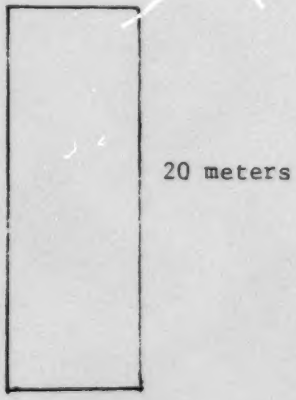
14. Find the area of ABCD _____



15. Find the area of the square, EFGH.

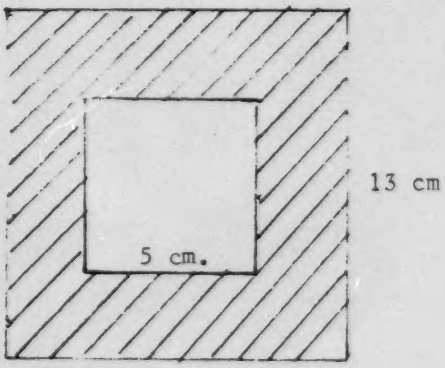


16. Find the area _____



17. The shape of the picture frame is a square with a square cut out.

Find the AREA of the picture frame. (Be very careful.)



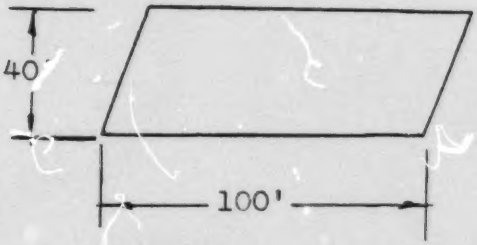
Lesson 5

Aim

To calculate the area of a parallelogram and a triangle

Motivation

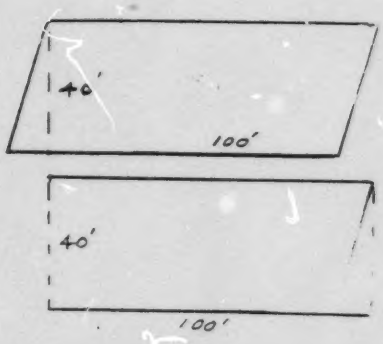
A builder buys a plot of land in the shape of a parallelogram, at a cost of $5 a square foot. What is the cost of the plot?



Materials

Parallelograms and triangles cut from heavy art paper

Development

Teacher Activity	Student Activity
1. Prepare a parallelogram to represent the plot of land. Indicate the height of the parallelogram with a dashed line. Then cut along this dashed line to form a triangle. Move the triangular piece to form a rectangle from the original parallelogram.	
2. Guide the students to recognize that the base of the parallelogram is equivalent to the base of the rectangle, while the height of the parallelogram is equivalent to the height of the rectangle.	2. Rectangle or Parallelogram: A = l x w or A = b x h.

3. Have the students solve the motivation problem.

3. $A = bh$

$A = 100 \times 40$

$A = 4000$

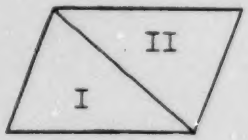
The area is 4000 square feet.

$$\begin{array}{r} 4000 \\ \times\ \$5 \\ \hline \$20000 \end{array}$$

The plot will cost $20,000.

4. Prepare two congruent(identical) triangles. Demonstrate that by placing them properly a parallelogram can always be formed. Since we now know the formula for the area of a parallelogram, we need to take one-half of it to determine the area of either triangle.

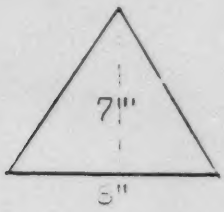
4.



Area of a triangle:

$A = \frac{1}{2}bh$ or $\frac{bh}{2}$

5. Have the students find the area of a triangle.



5. $A = \frac{bh}{2}$

$A = \frac{8 \times 7}{2}$

$A = 28$

The area is 28 in^2.

Summary

1. What is the formula for area of a parallelogram?
2. What is the formula for area of a triangle?

SUGGESTED EXERCISES

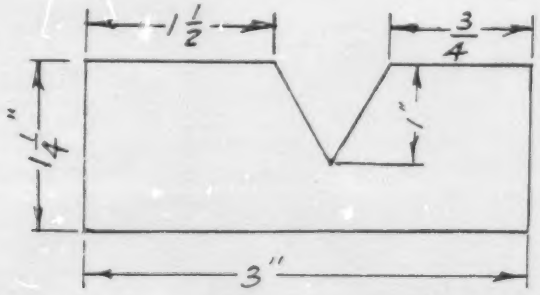
1. A parallelogram has a base of 35 cm and a height of 25 cm. Find its area.
2. Find the area of a triangle with a base of 4' and a height of 9'.

3. Find the area of each of the following figures:

PARALLELOGRAMS		TRIANGLES	
Base	Height	Base	Height
a. 9 in.	4 in	a. 7 in.	6 in.
b. 6 1/2 in.	3 1/2 in.	b. 14 ft	3 ft
c. 3 ft	10 in.	c. 20 in.	1 ft
d. 30 m	15 m	d. 15 m	7 m
e. 65 cm	35 cm	e. 25 cm	15 cm
f. 2.5 m	4 m	f. 12 m	2.5 m

4. A triangle has an area of 39 square inches. If its base is 12 inches, what is its height?

5. A "vee" slot is milled out of a steel block as shown. Calculate the remaining surface area.



UNIT V

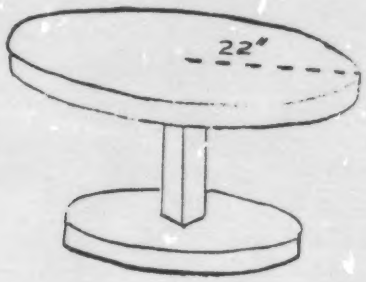
Lesson 6

Aim

To calculate the area of a circle

Motivation

What is the area of the circular dining room table top?

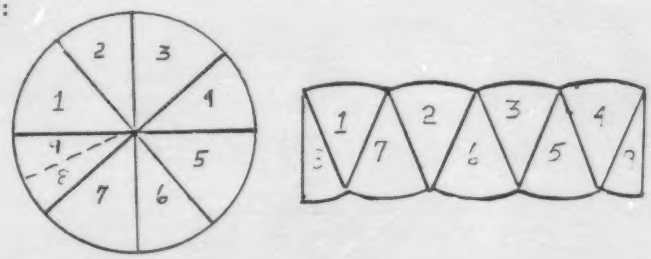


Materials

Overhead projector, prepared acetate

Development

Teacher Activity	Student Activity
1. Using the problem in the Motivation, discuss the need for a formula for the area of a circle.	
a. On a prepared overhead acetate, have available a circle divided into eight equal sectors, of which one is bisected.	
b. Move the sectors into a rectilinear rearrangement as illustrated:	



c. Guide students to realize that the length of the rectangle thus formed is equivalent to one-half the circumference of the circle, and its width is equivalent to the radius of the circle. By making the appropriate substitutions in the formula A = lw, the formula for area of a circle can be developed.

c. $A = lw$ But $l = C/2$ and $w = r$.

$A = \frac{C}{2} \times r$

Using $C = 2\pi r$,

$A = \frac{2\pi r}{2} \times r$

$A = \pi r \times r$

$A = \pi r^2$

d. What is the formula for the area of a circle?

d. $A = \pi r^2$

2. Have the students use the formula to solve the motivation problem.

2. $A = \pi r^2$

$A = 3.14 \times (22)^2$

$A = 3.14 \times (484)$

$A = 1519.76$

The area is about 1520 square inches.

Summary

1. What is the formula for area of a circle?

SUGGESTED EXERCISES

1. Find the area of each of the following circles: (use $\pi = 3.14$ or $\pi = 22/7$)

a. Radius = 7 inches

b. Radius = 10 feet

c. Radius = 14 meters

d. Radius = 20 yards

e. Radius = 5 inches

f. Radius = 14.7 cm

g. Diameter = 1 ft 8 in.

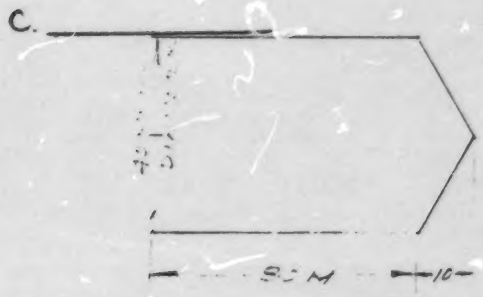
h. Diameter = 84 cm

2. Find the area of each of the following figures:

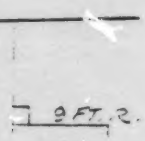
A. _____

B. _____

C.



D. _____

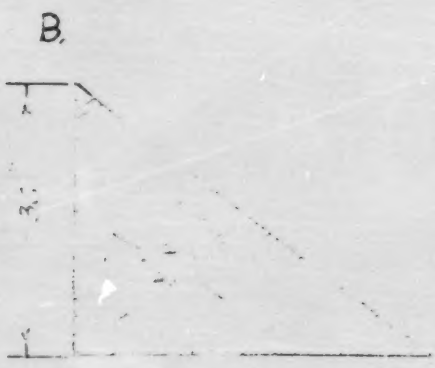


3. Find the area of the shaded portions in the diagrams shown below.

A.

A _____

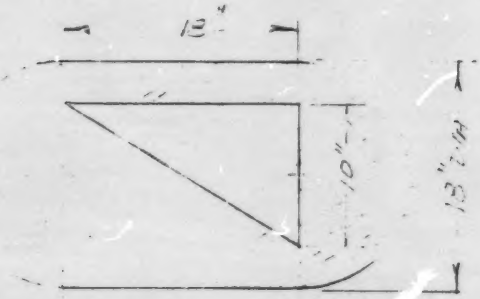
B.



F. _____

3 (Continued).

C.



D.

Lesson 7

Aim

To learn the concept of volume

Motivation

Carmen wants to mail fudge brownies, which have been cut into cubes, to her brother in San Francisco. How many can she place in a box?

Materials

Different size cube blocks, yardstick, meterstick

Development

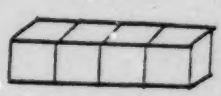
Teacher Activity	Student Activity
1. Using the problem in the motivation, discuss the fact that insufficient data has been given to solve the problem. (Size of cubes and box are not known.)	
2. Elicit that cubes of brownies and the box are three-dimensional figures (solids) involving length, width and height (depth).	
3. Have students review definition of Area as the measure of a region. Inform students that the measure of a solid is called volume.	
4. Elicit that just as a unit of length is used to measure a line segment and a unit of area is used to measure a region, a unit of space or solid is used to measure the volume of a solid.	
5. Have pupils suggest various unit solids that can be used to measure volume.	5. Rectangular solid, cubic solid, spherical solid, etc.

6. Lead pupils to see that the cubical solid is preferable.

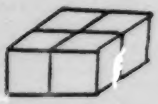
7. Have pupils suggest standard units of volume.

7. Cubic inch; cubic foot; cubic yard; cubic meter; etc.

8. Using the diagrams below, have the students determine the volume of each of the figures formed by one-inch cubes.

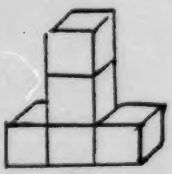


a.



b.

c.



d.

9. Return to the motivation question, and ask: "What size carton will be needed to box 24 fudge brownies, each a one-inch cube?"

10. Using one-inch cubes, have students experiment to see that several size boxes will hold the 24 brownies.

10. Among the possible sizes are:
1" x 4" x 6"
2" x 2" x 6"
2" x 3" x 4"

Summary

1. What kind of units are used to measure length?

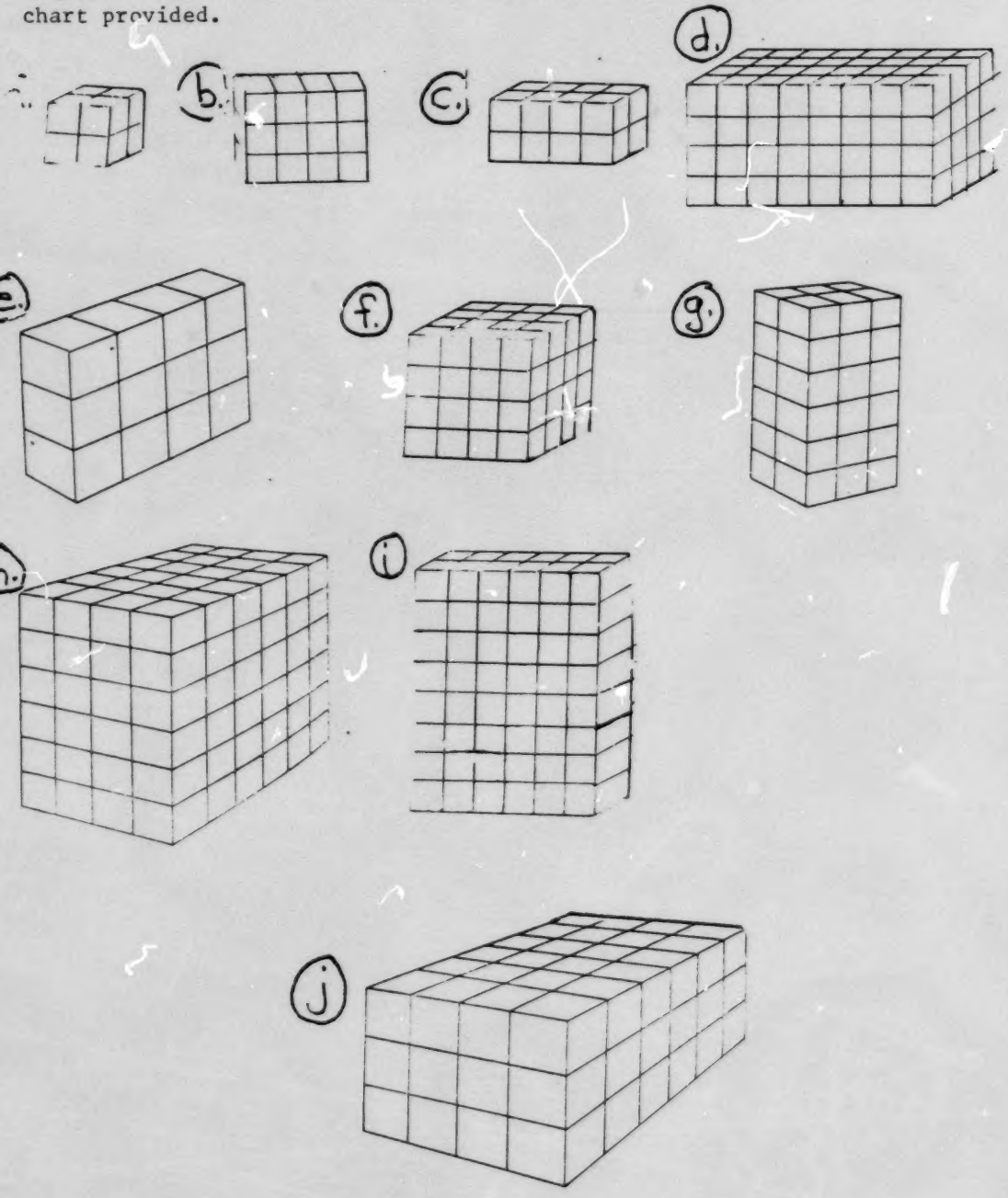
2. What kind of units are used to measure area?

3. What kind of units are used to measure volume?

4. How many dimensions does a solid figure have?

SUGGESTED EXERCISES

1. Count the number of cubic units in each solid. Record your data in the chart provided.



	WIDTH	HEIGHT	LENGTH	VOLUME
a.				
b.				
c.				
d.				
e.				
f.				
g.				
h.				
i.				
j.				

2. The crafts worker very often has to know the volume or interior space of a structure which is a rectangular solid in form.

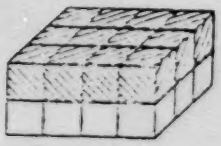


FIG 1

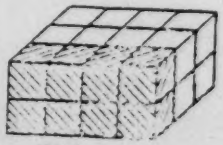


FIG 2

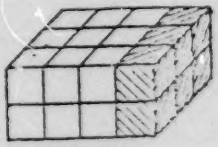


FIG 3

A RECTANGULAR SOLID

a. How many cubes are shaded in figure 1?
b. If each cube measured 1 cm x 1 cm x 1 cm, how many cubic centimeters are shaded in figure 1, figure 2, and figure 3?
c. Instead of counting cubes, how can we find the volume of the shaded geometric figure (length x width x depth)? Explain your conclusion.

UNIT V

Lesson 8

Aim

To calculate the volume of a rectangular solid using the formula: $V = l \times w \times h$

Motivation

A carton measures 5" long, 2" wide and 4" deep. What is the maximum number of boxes (one cubic inch in size) which would be placed in the carton?

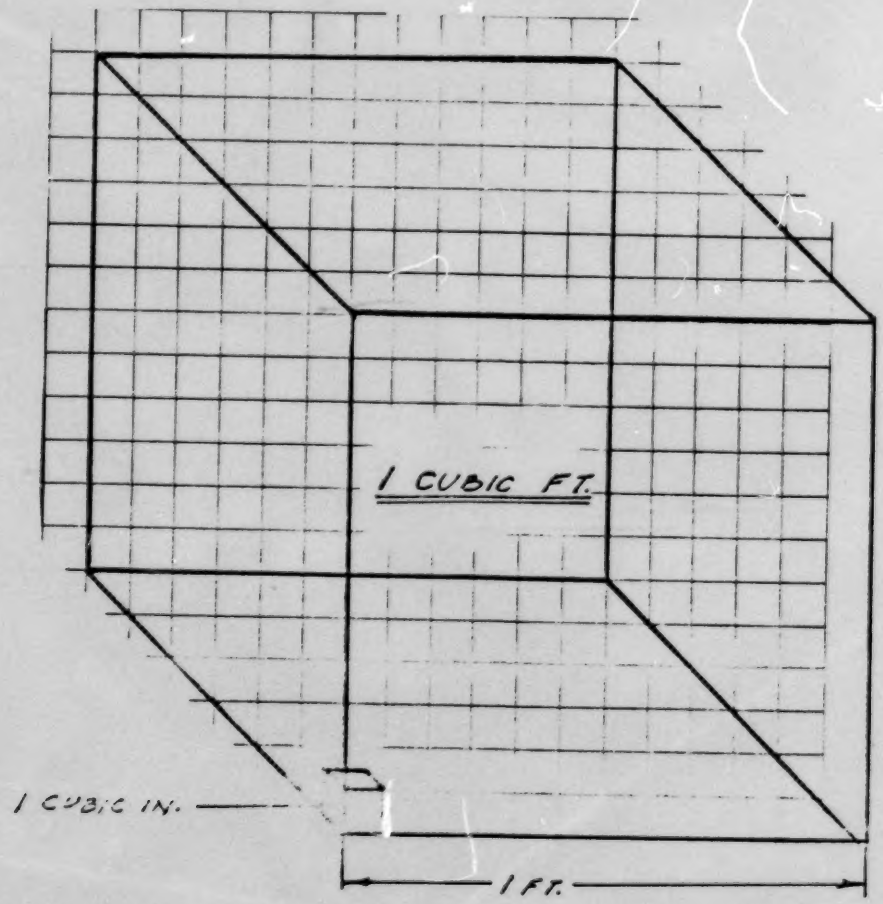
Materials

Carton, cubic inch blocks

Development

	Teacher Activity	Student Activity
1. a.	Using a carton with the dimensions listed above and cubic inch blocks, have students determine the volume of the carton.	
b.	Pose the question: Is there a more direct method of determining the volume?	b. Yes
c.	Elicit the relationship between the dimensions of the carton and its volume.	c. $= l \times w \times h$ $V = lwh$
d.	Inform the students that a three-dimensional solid, all of whose faces are rectangles, is called a rectangular solid. (Note to Teacher: Remind students that the same standard unit must be used for all three dimensions.)	

2. Have students show why 1 cu. ft. = 1728 cu. in.



3. Elicit that a cube is a rectangular solid, all of whose edges are equal in size.

4. Using the formula for the volume of a rectangular solid, have students derive the formula:
Volume of Cube = e^3
Where e = length of an edge.

4. V = lwh, and since
e=l=w=h,
V = eee, or,
Cube: V = e^3.

5. Have students use the formula for the volume of a cube to determine the number of cubic feet in a cubic yard.

(recall the alternate notations:
cu yd = yd^3

cu ft = ft^3)

5. e = 3 ft
$V = e^3$
$V = 3^3$
V = 27
1 cu yd = 27 cu ft
$1\ yd^3 = 27\ ft^3$

Summary

1. What is the formula for the volume of a rectangular solid?
2. What is the formula for the volume of a cube?

SUGGESTED EXERCISES

1. Find the volume of each rectangular solid described below:

	WIDTH	LENGTH	HEIGHT
a.	4 in	9 in.	2 in
b.	6 cm	10 cm	3 cm
c.	1½ in	1½ in	1½ in
d.	7.2 cm	4.5 cm	1.4 cm
e.	10 yd	10 yd	2 yd
f.	7.0 cm	10.3 cm	.7 cm
g.	4.7 m	6.5 m	4 m
h.	3 1/4 in	2 3/4 in	2 1/4 ft (careful)

2. Find the volume of a cubic carton, 14 inches on each side.

3. How many cubic feet of air are there in a classroom 30 feet long, 25 feet wide, and 10 feet high?

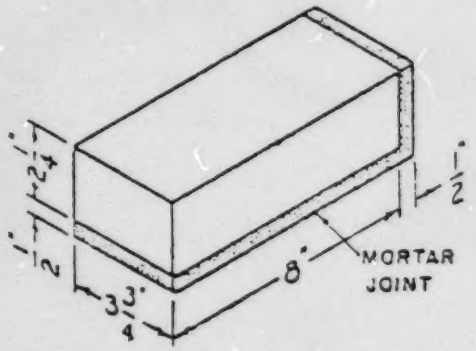
4. How many cubic feet of water will fill a swimming pool that is 20 ft by 10 ft, to a depth of 8 ft?

5. In order to install a water main, a trench must be dug which is 85 ft. long, 3 feet wide, and 4 feet deep. How many cubic yards of earth must be excavated?

6. A standard common brick is 8" long, 3-3/4" wide, and 2-1/2" high.

   a. How many cubic inches does the brick contain?

   b. If 1/2" mortar is used for the bed joint, how many cubic inches does the unit (brick and mortar) contain?



STANDARD COMMON BRICK

7. Measure the size of your kitchen (length, width, and height) and calculate its volume in cubic feet. Convert your answer to cubic inches.

Lesson 9

Aim

To calculate the volume of a circular cylinder

Motivation

A cylindrical container has a radius of three inches and a height of seven inches. How many cubic inches of gasoline can it hold?

Materials

Several disks with the same diameter and thickness

Development

Teacher Activity	Student Activity
1. a. Elicit that the formula for volume of a cylinder must be obtained.	1.
b. Using cylindrical disks of the same size, develop the concept that we can think of a cylinder as being composed of a series of circular disks. If each disk is one unit high, the number of cubic units it contains will be numerically equal to the number of square units it contains. Therefore, the volume of the cylinder can be obtained by determining the product of the height and the area of the base.	b. Volume of a cylinder = area of circular base multiplied by the height. Therefore, V = (area of base) x h, $V = \pi r^2 h$
c. Have the students compute the volume of the container in the motivation problem.	c. $V = \pi r^2 h$ $V = \frac{22}{7} \times 3^2 \times 7$ $V = \frac{22}{7} \times 9 \times 7$ V = 198 The container holds 198 cubic inches.

2. Inform the students that 231 cubic inches equals one gallon.	2. The container holds slightly less than one gallon of gasoline.

Summary

1. What is the formula for finding the volume of a cylinder?

SUGGESTED EXERCISES

1. Find the volume of a cylinder whose radius is 10 cm, and whose height is 30 cm.

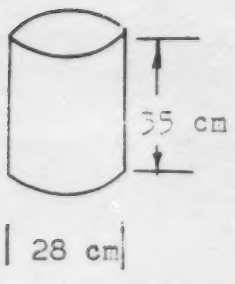
2. A cylindrical tank is 40 feet high and has a diameter of 28 feet. What is its volume?

3. Compare the volumes of these two cylinders:
   a. radius = 7 inches, height = 15 inches.
   b. radius = 7 inches, height = 30 inches.

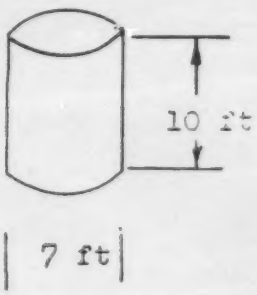
4. Compare the volumes of these two cylinders:
   a. radius = 10 inches, height = 14 inches.
   b. radius = 20 inches, height = 14 inches.

5. Find the volume of each of these cylinders:

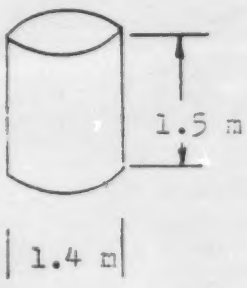
a.



b.



c.



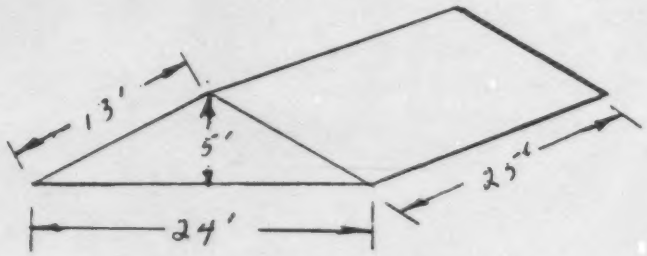
Lesson 10

Aim

To calculate the volume of a triangular prism

Motivation

A gable roof has the following dimensions:



In order to vent this attic properly, we must determine the number of cubic feet of air it contains. How can the volume of the attic be determined?

Materials

Triangular prism, congruent triangular blocks.

Development

Teacher Activity	Student Activity
1. a) Show the students a model of a triangular prism. Discuss the shape of its faces and its name.	1. a) A solid figure whose two bases are triangles, and whose sides (lateral faces) are rectangles, is called a triangular prism. (Note that the ends of the attic would be considered the bases of the triangular prism.)
b) Elicit that a formula for finding its volume must be found.	
2. a) Have the students consider the previously developed formulas for the volume of a rectangular solid and a cylinder. Elicit that each involves multiplying the area of the base by the height.	2. a) Rectangular solid: $V = lwh$ Cylinder: $V = \pi r^2 h$ Volume = (area of base) x h

b) By stacking a set of triangular blocks, guide the students to develop the general formula: $V = Bh$.

3. a) Have the students use this newly developed formula to solve the problem presented as Motivation.

b) What is the shape of the attic?

c) What are the dimensions of its base?

d) What is the formula for area of a triangle?

e) What is the volume of the attic?

b) For a triangular prism, the volume equals the area of the base (B) multiplied by the height.

$V = Bh$

(Note the difference between B, the area of the triangular base, and b, the base of the triangle.)

3.

b) The attic is a triangular prism or solid.

c) The triangle has a base of 24 ft and a height of 5 ft.

d) $A = \frac{bh}{2}$

e) $V = B\,h$

$V = \frac{24 \times 5}{2} \times 25$

$V = 60 \times 25$

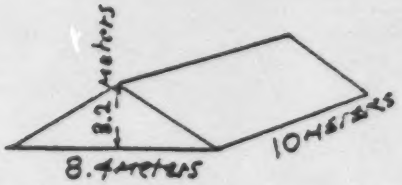
Volume = $1500\ ft^3$

Summary

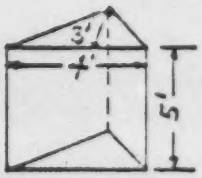
1. What is the formula for the volume of a triangular prism?

SUGGESTED EXERCISES

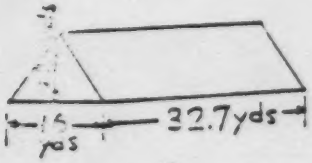
1. Find the volume of the following triangular prisms.



A

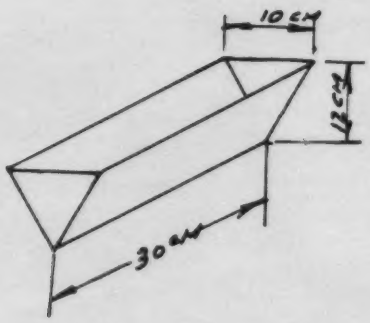


B



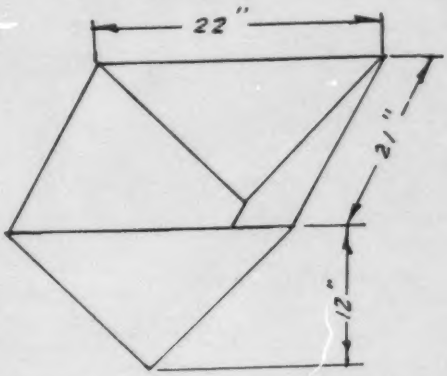
C

2. How many cubic centimeters of water will the trough hold when it is filled completely?

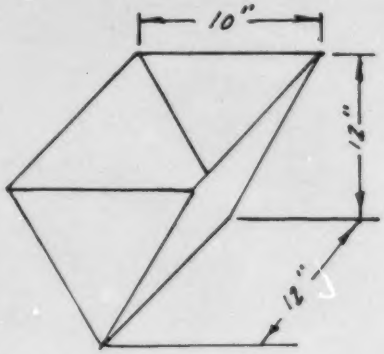


3. Find the volume of a triangular prism whose altitude is 6-1/4 inches and whose base is a right triangle with legs 4-1/2" and 3-3/4".

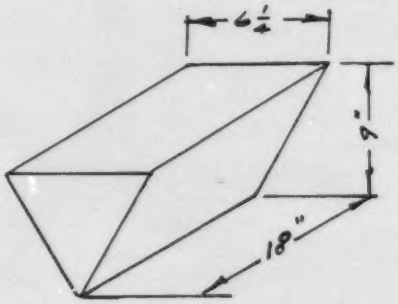
4. Which of the following three troughs, when filled completely, will hold the most water, and which will hold the least amount of water?



A.



B.



C.

Lesson 11

Aim

To calculate the surface area of a solid

Motivation

A rectangular metal block, 15" x 12" x 10" is to be completely covered with metallic paint. How many square inches of surface must be covered by the paint?

Materials

A carton box; a cube; a tin can and a pair of metal snippers

Development

Teacher Activity	Student Activity
1. a) Using a carton box as a model, elicit that it will be necessary to find the total area of the six sides of the block. Inform students that this is called the surface area of a figure.	1.
b) How can we find the area of the face that is 15" x 12"?	b) $A = lw$ $A = 15 \times 12$ Area is 180 in^2
c) How many faces of this size are there?	c) Two. 2 x 180 in^2 = 360 in^2
d) How can we find the areas of the remaining faces?	d) 12 in. x 10 in. = 120 in^2 2 x 120 in^2 = 240 in^2 15 in. x 10 in. = 150 in^2 2 x 150 in^2 = 300 in^2
e) What is the total surface area of the metal block?	e) 360 240 300 900 The surface area is 900 square inches.

f) Have the students relate the numbers used in the computation to the terms length, width, and height. Using this relationship, have them express the procedure in terms of a formula.

f) Surface area of a rectangular solid:
$S = 2lw + 2lh + 2wh$

or

$S = 2(lw + lh + wh)$

2. a) Pose the problem: What is the surface area of a cube with dimensions 5" x 5" x 5" ?

2. a) Each of the six faces has the same area, 25 square inches. Therefore, the surface area is 6 x 25 in^2 or 150 in^2.

b) Guide students to realize that this can be expressed as a formula in terms of 'e', the length of each edge.

b) $S = 6e^2$

3. a) Show the students a can. Challenge them to find the surface area to be coated with tin. Discuss the importance of cylindrical shapes in industry, trade, and consumer goods.

3. a) Name as many industrial and consumer items as you can that have a cylindrical shape.

b) Demonstrate the composition of a cylinder by cutting a tin can to show two congruent circles and a rectangle. Guide students to realize that the length of the rectangle is equal to the circumference of the original cylinder, and the width of the rectangle is equal to the height of the cylinder.

c) What will be measured to find the total surface area of the cylinder?

c) We will need to measure the area of the two circles and the rectangle.

d) Using the formulas $A = \pi r^2$ and $C = 2\pi r$, guide the students to develop a formula for the surface area of a cylinder.

$S = 2\pi r^2 + 2\pi rh$

d) Area of one base: πr^2

Area of both bases: $2\pi r^2$

Area of rectangle = $l \times w$
= $C \times h$
= $2\pi rh$

Bases + rectangle=
$2\pi r^2 + 2\pi rh$

4. Have the students recall the Distributive Property of Multiplication over Addition:
$5x3 + 5x4 = 5(3 + 4)$
Guide them to realize that the factor "$2\pi r$" is common to both terms of the new formula.

4. $S = 2\pi r^2 + 2\pi rh$ can also be written as $S = 2\pi r(r + h)$.

5. Have the students find the surface area of a cylindrical can whose radius is two inches, and whose height is eight inches.

5. $S = 2\pi r(r + h)$
$S = 2(3.14)(2)(2+8)$
$S = 12.56(10)$
$S = 125.6$
The surface area is 125.6 square inches.

Summary

1. What do we mean by the surface area of a solid?
2. What is the formula for the surface area of a cube?
3. The formula $S = 2\pi r(r+h)$ equals the surface area for what geometric figure?

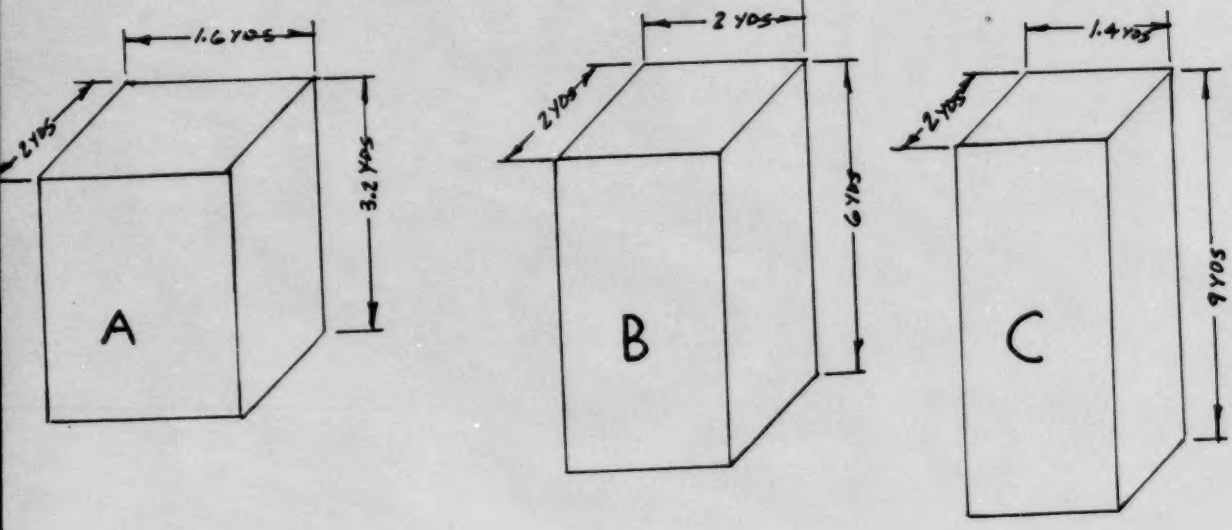
SUGGESTED EXERCISES

1. Find the surface area for rectangular solids with these dimensions:

a.	Length: 20 inches	Width: 18 inches	Height: 15 inches
b.	Length: 5 feet	Width: 4 feet	Height: 3 feet
c.	Length: 15 meters	Width: 10 meters	Height: 6 meters
d.	Length: 2 yards	Width: 2 yards	Height: 2 yards

2. Find the surface area of a cylindrical tank whose radius is 15 feet, and whose height is 40 feet.

3. Using the following concrete columns, which has the most surface area and which has the least surface area? How many cubic yards of concrete would be required to construct each of these columns if each column is 2 yards deep?



Lesson 12

Aim

To introduce the units used in liquid measurement

Motivation

A fish tank is 14 inches long, 10 inches wide, and 10 inches deep. What is its volume? How many gallons of water will it hold?

Development

Teacher Activity	Student Activity
1. a) Have the students consider the problem in the Motivation.	1. a) For a rectangular solid: $V = lwh$ $V = 14 \times 10 \times 10$ $V = 1400$ The tank has a volume of 1400 cubic inches.
b) Elicit that the number of cubic inches in one gallon must next be determined. Inform the students that 231 cubic inches equals one gallon.	b) $231\ in^3 = 1\ gal$ $231\overline{)1400}$ R 14 The tank will hold slightly more than six gallons.
2. a) Pose this problem: How many gallons can be stored in a one-cubic-foot container? (Students should refer to their list of Customary Units.)	2. a) $1728\ in^3 = 1\ ft^3$ $231\ in^3 = 1\ gal$ $231\overline{)1728.00}$ R $7.48...$ $1\ ft^3 = 7.5\ gal$, (to the nearest tenth)
3. How many gallons of water are needed to fill a swimming pool 15 ft long, 10 feet wide, to a depth of 6 feet?	3. $V = lwh$ $V = 15 \times 10 \times 6$ $V = 900\ ft^3$ $900 \times 7.5 = 6750$ The pool will need 6750 gallons of water.

4. a) Have the students find the volume of a cubic container each of whose edges is ten centimeters in length.

b) Inform the students that the symbol "cc" is often used to represent cubic centimeters, although "cm^3" is preferable.

c) Inform the students that 1000 cubic centimeters is equivalent to one liter. The liter is the basic unit of liquid measure in the metric system and is a little larger than one quart.

4. a) $V = e^3$

$V = 10^3$

$V = 1000$

The volume is 1000 cubic centimeters.

The volume is 1000 cm^3 or 1000 cc.

c) 1 liter = 1000 cm^3

1 liter = 1000 ml

Therefore, 1 ml = 1 cm^3

5. Have the students calculate the number of liters of paint a rectangular spray can can hold when filled to capacity, if its dimensions are 25 cm by 30 cm by 45 cm.

5. $V = lwh$

$V = 25x30x45$

$V = 33{,}750\ cm^3$

$\frac{33{,}750\ cm^3}{1000\ cm^3}$ (1 liter) =

33.75 liters

Summary

1. How many cubic inches equal one gallon?

2. Approximately how many gallons of water are contained in one cubic foot?

3. How many cubic centimeters make one liter?

SUPPLEMENTARY EXERCISES

1. John mixed 650 ml of water with 350 ml of frozen juice concentrate. How many liter(s) of liquid did he have?

2. Carmen had to divide two liters of motor oil into five equal portions. How many milliliters of oil were there in each portion?

3. Sam wants to make a vinyl cement. He needs 1500 ml of water for a given volume of concentrate. If he wants to make six times the amount, how many liters of water should he use?

4. A fuel mixture calls for 400 ml of gasoline and 100 ml of alcohol. When these quantities are mixed, what is the volume of fuel, expressed in liters?

5. Eric had 2.6 liters of propane gas. If he had to divide it into four equal parts, how many milliliters of propane would there be in each part?

6. In order to remove 8640 cubic feet of earth from an excavation, a contractor used a truck with a capacity of six cubic yards. How many trips would the truck have to make to complete the job?

7. Estimate the length, width, and height of your classroom. Calculate the number of cubic feet it contains. How many cubic yards is that?

8. A 15-gallon fish tank contains about how many cubic feet?

Unit VI

FORMULAS AND EQUATIONS

Lesson 1 To evaluate arithmetic expressions using the rules for order of operations
(GHSM p. 11)*

Lesson 2 To translate and evaluate algebraic expressions
(GHSM pp. 14-15)*

Lesson 3 To evaluate algebraic expressions involving powers

Lesson 4 To write a formula from a verbal sentence
(GHSM p. 37)*

Lesson 5 To evaluate formulas used in common trades
(GHSM pp. 40, 42, 65)*

Lesson 6 To solve equations of the form $x + a = b$ and $x - a = b$, using the addition and subtraction properties of equalities
(GHSM pp. 11, 12, 15, 17, 27)*

Lesson 7 To solve equations of the form $ax = b$ and $x/a = b$, using the multiplication and division properties of equalities
(GHSM pp. 15, 17, 26, 27)*

Lesson 8 To solve equations of the form $ax + b = c$ and $ax - b = c$
(GHSM p. 27)*

*General High School Mathematics, New York State Department of Education, August 1978

Lesson 1

Aim

To evaluate arithmetic expressions using the rules for order of operations

Motivation

Perform the indicated operations: $7 + 3 \times 2$.

Development

Teacher Activity	Student Activity
1. Elicit that the expression might be interpreted in two ways. (It is said to be ambiguous.) Therefore, mathematicians have agreed to perform operations according to a set of rules.	1. $7 + 3 \times 2$ might equal 10×2 or 20. $7 + 3 \times 2$ might equal $7 + 6$ or 13.
2. Inform students that, when more than one operation is indicated in an expression, the following order of operations must be followed:	
a) Perform multiplication and division, in order, from left to right.	
b) Then perform addition and subtraction, in order, from left to right.	
3. Have the students use the rules for order of operations to solve the problem in the motivation.	3. $7 + \underbrace{3 \times 2}_{6} =$ $7 + 6 =$ 13

4. Have students solve each of the following:

a) $9 + 10 + 2 = ?$

b) $6 \times 12 + 3 \times 7 = ?$

4.

a) $9 + 10 + 2 =$
$9 + 5 =$
14

b) $6 \times 12 + 3 \times 7 =$
$72 + 21 =$
93

5. a) Pose the problem: Irene is paid $4 an hour. If she works three hours Friday and seven hours Saturday, how much does she earn?

b) Elicit the need to add <u>first</u> in this problem.

c) Inform students that parentheses are used as a grouping symbol. The operations indicated within parentheses are performed first.

5. a) Some students may write: $4 \times 3 + 7 = 19$. Others may write: $3 + 7 \times 4 = 31$. But, in working ten hours, Irene earned $40.

c) 1. $(3 + 7) \times 4 =$
$(10) \times 4 =$
40

2. $(3 + 7) \times 4$ is usually written $(3 + 7)4$.

6. Have the students evaluate each expression, using the rules for order of operations:

a) $7 + 3 \times 6$

b) $(7 + 3)6$

c) $8 - 3 - 2$

d) $8 - (3 - 2)$

6.

a) $7 + 3 \times 6 =$
$7 + 18 = 25$

b) $(7 + 3)6 =$
$(10)6 = 60$

c) $8 - 3 - 2 =$
$5 - 2 = 3$

d) $8 - (3 - 2) =$
$8 - (1) = 7$

<u>Summary</u>

1. If an expression involves both addition and division, which operation is performed first?

2. If there are operations indicated both inside and outside parentheses, which are performed first?

SUGGESTED EXERCISES

1. Evaluate each of these expressions:

a) $9 \times 2 + 3 \times 4$

b) $5 + 3 \times 6$

c) $16/4 + 8$

d) $14 - 4 \times 2$

e) $14 - 4 - 2$

f) $17 - 8 \times 3$

g) $26 - 12/2$

h) $6 + 8 \times 3$

i) $26 - 14 - 12 + 2$

j) $18 + (6 - 3)$

k) $16 - (4 + 3)$

l) $10(5 + 9)$

m) $0.5(7 + 9)$

n) $(8 - 6) \times (5 + 1)$, or $(8 - 6)(5 + 1)$

o) $20 - (3 - \frac{1}{2})$

p) $(8 + 2 \times 5) + 3 \times 5$

q) $16/4 - (6 + 15/3)$

r) $9 - (14 - 5)$

2. Find the value of each expression:

a) $6 + 3 \times 2 + 1$

b) $(6 + 3)2 + 1$

c) $(6 + 3)(2 + 1)$

d) $6 + 3(2 + 1)$

e) $6 + 5 \times 3 - 2$

f) $(8 + 5)(3 - 2)$

g) $8 + 5(3 - 2)$

h) $(8 + 5)3 - 2$

i) $4 + 3 \times 2 - 5$

j) $(4 + 3)2 - 5$

k) $4 + 3(2 - 5)$

l) $(4 + 3)(2 - 5)$

m) $4 \times 2 + 3 \times 4$

n) $4(2 + 3)4$

o) $4(2 + 3 \times 4)$

p) $(4 \times 2 + 3)4$

Lesson 2

Aim

To translate and evaluate algebraic expressions

Motivation

Which of these are sentences?

a) the little hungry child
b) She saw it clearly.
c) only a cup of rice
d) Chuck ate two hot dogs.

Development

Teacher Activities	Student Activities
1. Discuss the motivation problem. Inform the students that "the little hungry child" and "only a cup of rice" are called <u>expressions</u>.	1. "She saw it clearly." and "Chuck ate two hot dogs." are sentences.
2. Have the students consider the difference between mathematical sentences and mathematical expressions. These are expressions: a) $5 + 3$ b) $7 - (4 + 3)$ c) $8 + x$ These are sentences: a) $5 + 2 = 7$ b) $8 > 4$ c) $6 < 8 \times 8$ d) $5 + t = 9$	2. A mathematical sentence contains a symbol of equality or inequality. A mathematical expression contains no such symbol.
3. Pose the question: How can we write the sentence "Five times some number equals 135."?	3. $5n = 135$ (Note that $5n$ is used instead of $5 \times n$ to avoid ambiguity.)
4. Have the students recall that letters are often used to represent numbers in mathematical expressions and sentences.	

5. Inform students that a letter which is used to represent a number is called a <u>variable</u>.

6. Have the students translate verbal phrases into algebraic language, using the variable 'n' to represent each number.

   a) six more than a number
   b) four times a number
   c) nine decreased by a number
   d) one less than a number
   e) four divided by a number
   f) a number divided by ten

6.

   a) 6 + n or n + 6
   b) 4n
   c) 9 - n
   d) n - 1
   e) 4/n
   f) n/10

7. Inform the students that we can evaluate an expression only if we know the value of any variables it contains.

8. Have the students evaluate simple expressions.

   a) What is the value of n + 6 if n = 4 ?
   b) Find the value of 3n - 2 when n = 6.

8. Substitute numbers for variables:

   a) n + 6
      (4) + 6 = 10
   b) 3n - 2
      3(6) - 2 =
      18 - 2 =
      16

9. Introduce the use of a table of values:

n	5n + 6
1	
3	
5	
6	

9.

n	5n + 6
1	11
3	21
5	31
6	36

<u>Summary</u>

1. What is a variable?
2. Is "two is less than five" an expression or sentence?
3. Is "two less than five" an expression or sentence?
4. What is the value of 10n, if n equals seven?

SUGGESTED EXERCISES

1. Use mathematical symbols to translate each of these verbal expressions into an algebraic expression. Let x represent the number.

   a) a number decreased by five

   b) twelve decreased by a number

   c) the product of seven and a number

   d) the quotient of a number and six

   e) double a number

   f) twice the sum of three and a number

   g) one-half of a number

   h) a number increased by eight

   i) twice a number divided by three

   j) the product of a number and four

2. Complete each table of values for the indicated values of the variable.

a)

n	n - 2
6	
4	
2	
0	
-2	

b)

n	4n
3	
2	
1	
0	
-1	

c)

n	3n + 2
0	
1	
2	
3	
4	

d)

x	8 - x
0	
3	
6	
9	
-3	

e)

x	x/4
0	
2	
4	
6	
-2	

f)

x	3 + 2x
0	
1	
2	
3	
-3	

Lesson 3

Aim

To evaluate algebraic expressions involving powers

Motivation

What is the value of each of the following?

a) 6×6

b) $5 \times 5 \times 5$

c) $3 \times 3 \times 3 \times 3$

d) $2 \times 2 \times 2 \times 2 \times 2$

Development

Teacher Activity	Student Activity
1. Using the motivation, review the definitions of product, factor, and repeated factor.	1. a) 36 b) 125 c) 81 d) 32
2. Have students recall that when a factor is repeated, the number of times it is repeated can be represented by an <u>exponent</u>.	2. a) $6 \times 6 = 6^2$ b) $5 \times 5 \times 5 = 5^3$ c) $3 \times 3 \times 3 \times 3 = 3^4$ d) $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
3. How do we read: a) 6^2 b) 5^3 c) 3^4	3. a) 6^2 is read as "six squared" or "six to the second power". b) 5^3 is read as "five cubed" or "five to the third power". c) 3^4 is read as "three to the fourth power".
4. What does n^4 mean? Recall that: a. the n is called the <u>base</u>: b. the 4 is called the <u>exponent</u>: c. the term n^4 is called the <u>power</u>.	4. It means $n \cdot n \cdot n \cdot n$ $\text{base}^{\text{exponent}} = \text{power}$

5. What is the exponent of "5" ?

6. Have the students evaluate expressions involving powers:
   a) e^3 when e = 5
   b) $5n^2$ if n = 4
   c) $(5n)^2$, if n = 4

5. It is understood to be one.
   $5 = 5^1$

6. a) e^3
   $(5)^3 = 5 \times 5 \times 5 = 125$
   b) $5n^2$
   $5 \cdot 4^2 = 5 \times 4 \times 4 = 80$
   c) $(5x4)^2 = (20)^2 = 400$

Summary

1. In the expression "five squared", what is the exponent?
2. What is the value of "five squared"?
3. In the expression "n cubed", what is the exponent?
4. What is the value of "n cubed" when n equals two?

SUGGESTED EXERCISES

1. Express each in a shorter form, using exponents:

   a) $2 \cdot 2 \cdot 2$
   b) $a \cdot a$
   c) $5 \cdot 5 \cdot m \cdot m$
   d) $7 \cdot 7 \cdot 7 \cdot r \cdot r$
   e) $(2a)(2a)(2a)$
   f) $5 \cdot 8 \cdot 8 \cdot 8$

2. Evaluate each of the following when n = 3

   a) $7n^2$
   b) $5n^3$
   c) n^4
   d) $(2n)^2$
   e) $(3n)^2$
   f) $(2n)^3$
   g) 2^n

3. Evaluate each of the following, using the values
   a = 2 b = 3 x = 4 y = 5

   a) $b^2 + x^2$
   b) $y^2 - a^3$
   c) $9a - a^2$
   d) $x^2 + 2x + 3$
   e) $(xy)^2$
   f) $(ax)^2 + (by)^2$
   g) $(bx)^a$

4. Complete this chart.

POWER	BASE	EXPONENT	MEANING	VALUE
2^3	2	3	2x2x2	8
4^2				
6^3				
9^2				
7				
8^2				
10^3				
10^6				
10^9				
1^{25}				
0^5				

Lesson 4

Aim

To write a formula from a verbal sentence

Motivation

This table shows the relationship between the pitch of a spur gear (p), the number of teeth (n) in the gear, and the diameter of the gear (d). Find the missing values.

n	10	20	30	40	60
d	4"	8"	6"	3"	12"
p	40	160	180		

Development

Teacher Activity	Student Activity
1. Have the students find the missing values.	1. n = 40 d = 3" p = 120 n = 60 d = 12" p = 720
2. Ask the students to express this relationship in the form of a mathematical rule.	2. The pitch (p) equals the number of teeth (n) multiplied by the diameter (d) of the gear.
3. Have students recall that mathematical rules can be shortened and written as formulas.	3. p = nd, or nd = p
4. Review the steps used in evaluating a formula.	4. a) Write the formula. b) Substitute appropriate values. c) Solve for the unknown. d) Write and check the answer.
5. Present a variety of mathematical rules and have the students express them as formulas.	

Summary

1. To find the perimeter of a triangle, we measure the three sides and add their lengths. Express this as a formula.

2. What are the steps we follow in evaluating a formula?

SUGGESTED EXERCISES

1. Express each as a formula:

   a) The sum of the angles A, B, and C, in a triangle, is 180°.

   b) The area of a circle (A) is equal to the product of pi and the square of the diameter (d), divided by four.

   c) The displacement of a piston (D) equals the area of the piston (A) times the stroke (s).

   d) The pitch of a roof (p) is equal to the rise (r) divided by the span (s).

   e) The amount of work done (W) is equal to the product of the force exerted (F) and the distance (D) the object is lifted.

   f) The voltage (E) is equal to the number of amperes (I) multiplied by the resistance (R).

   g) The number of screw threads per inch (p) is equal to one divided by the pitch (P).

   h) The area of an ellipse equals the product of pi, one-half the longer diameter (D), and one-half the smaller diameter (d).

2. Translate each of the following into an English sentence:

   a) $A = lw$

   b) $V = lwh$

   c) $d = rt$ (distance formula)

   d) $A = s^2$

   e) $A = \frac{1}{2}bh$

   f) $V = e^3$

   g) $A = \frac{1}{2}h(b_1 + b_2)$

Lesson 5

Aim

To evaluate formulas used in common trades

Motivation

A cylindrical tank, eight feet long and two feet in diameter, must be covered by insulation on all sides. How much insulation is needed?

Development

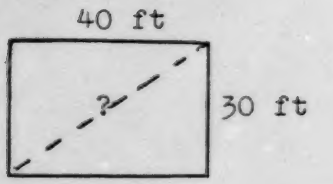
Teacher Activity	Student Activity
1. Elicit the need to determine the total surface area of the tank, and therefore the need to find the appropriate formulas.	1. $S = 2\pi r(r + h)$, and $d = 2r$
2. While solving, review the steps used in evaluating formulas, and the rules for order of operations.	2. $S = 2\pi r(r + h)$ (Write the formula) $S = 2(3.14)(1)(1 + 8)$ (Substitution) $S = 2(3.14)(1)(9)$ (Parentheses first) $S = 6.28(9)$ (Computation) $S = 56.52$ To cover the tank, 56.52 ft^2 of insulation is needed.
3. Supplement the lists of formulas already given in previous units with other formulas appropriate to the needs of your school and your students.	3. Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$ Interest formula: $I = prt$ Ohm's Law: $E = IR$ Distance formula: $d = rt$ Pythagorean Formula: $a^2 + b^2 = c^2$

4. Pose the problem:

A tunnel must be dug from one corner of a building, 40 feet by 30 feet, to the diagonally opposite corner, for a new drainage system. How long will the tunnel be?

(Review concept of square root, if necessary.)

4.



$a^2 + b^2 = c^2$

$(40^2) + (30^2) = c^2$

$1600 + 900 = c^2$

$2500 = c^2$

$50 = c$

The tunnel will be 50 ft.

Summary

1. What are the steps we follow in solving a problem through the use of a formula?

SUGGESTED EXERCISES

1. A room is 24 feet long and 10 feet wide. A wire is to be strung diagonally across it. How long will the wire be?

2. The formula for the length of the circumference of a circle can be used to compute surface speeds. The circumference multiplied by the revolutions per minute equals the surface speed. The relationship can be expressed as:

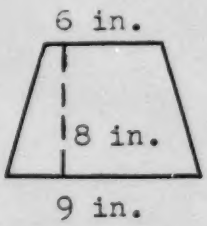
   S = C x (rpm), where,

   S = surface speed, C = circumference = $2\pi r$, ($\pi = 3.14$),

   and rpm = revolutions per minute.

   Using this formula, calculate the surface speed of a grinding wheel 8" in diameter (careful) turning at 1750 rpm. Express your solution to the nearest whole number.

3. Find the area:



Lesson 6

Aim

To solve equations of the form $x + a = b$ and $x - a = b$, using the addition and subtraction properties of equalities

Motivation

A carpenter wants to determine the width of a rectangular board. He knows that the sum of the width and ten inches is 25 inches. How can he determine the width?

Development

Teacher Activity	Student Activity
1. Have the students represent the relationship stated in the problem as an open sentence, using the variable 'w' to represent the width.	1. Let w = the width in inches Then, $w + 10 = 25$
2. Recall that a mathematical sentence which states that two quantities are equal is called an equation. Elicit that the open sentence can be changed to a true statement if we replace the variable 'w' with the number 15. The value of a variable which makes an open sentence true is called the solution.	2. $w + 10 = 25$ is an equation. $(15) + 10 = 25$ Therefore, $w = 15$
3. Discuss the need for a standard procedure to follow when an equation is more difficult to solve. Guide students to realize that the same number can be added to each side of an equation without affecting its equality.	3.a) $8 + 4 = 15 - 3$ $8 + 4 + (3) = 15 - 3 + (3)$ $15 = 15$ b) $8 + 4 = 15 - 3$ $8 + 4 + (-5) = 15 - 3 + (-5)$ $7 = 7$

4. Elicit that the equation from the motivation problem can be solved by adding -10 to each side of the equation.
Have the students solve the equation and check it using the principle of substitution.

4.
$$\begin{array}{rrcr} w & + 10 & = & 25 \\ & - 10 & = & -10 \\ \hline w & & = & 15 \end{array}$$

Check

$$\begin{array}{rrcr} w & + 10 & = & 25 \\ (15) & + 10 & \stackrel{?}{=} & 25 \\ & 25 & = & 25 \end{array}$$

5. Have the students solve and check: $x - 5 = 30$

5.
$$\begin{array}{rrcr} x & - 5 & = & 30 \\ & + 5 & & + 5 \\ \hline x & & = & 35 \end{array}$$

Check

$$\begin{array}{rrcr} x & - 5 & = & 30 \\ (35) & - 5 & \stackrel{?}{=} & 30 \\ & 30 & = & 30 \end{array}$$

Summary

1. What do we mean by the solution of an equation?

2. Does adding the same number to both sides of an equation affect the equality? Discuss.

[illegible]

1. Solve and check:

a) $x + 12 = 19$

b) $y - 10 = 16$

c) $x + 5 = -7$

d) $m - 28 = -19$

e) $a - 13 = -20$

f) $t + 17 = 12$

g) $y + 2 = -36$

h) $s + [illegible] = 11$

i) $b + [illegible] = 10$

j) $y - 54 = 6$

k) $t + 8 = 8$

l) $4 + y = -1$

Lesson 7

Aim

To solve equations of the form $ax = b$ and $\frac{x}{a} = b$, using the multiplication and division properties of equalities

Motivation

A mason determines that 15 square yards of a patio must be covered with stones. If the patio is 3 yards long, what is its width?

Development

Teacher Activity	Student Activity
1. a) Elicit the need to use the formula for the area of a rectangle.	1. a) $A = lw$
b) Have the students substitute the given data.	b) $15 = 3w$
c) Using trial and error, students will find the solution Check.	c) $5 = w$ or $w = 5$ Check. $15 = 3w$ $15 = 3(5)$ $15 = 15$
2. a) Elicit the desirability of developing a procedure for solving more difficult equations.	2.
b) Have students compare the original equation with the open sentence $5 = w$ to discover that the solution can be obtained by dividing both sides of the equation by three.	b) $15 = 3w$ $5 = w$ $\frac{15}{3} = \frac{3w}{3}$ $5 = w$
c) Use several other examples, leading students to discover the division property of equalities.	c) When both sides of an equation are divided by the same non-zero quantity, the equality is not affected.

3. a) In a similar way, develop the procedure for solving equations of the form $x/a = b$, using the multiplication property of equalities. (When equal quantities are multiplied by equal quantities, the results are equal.)

3.

b) Solve and check:

$\frac{x}{3} = 4$

b) $\frac{x}{3} = 4$

$3 \cdot \frac{x}{3} = 3 \cdot 4$

$x = 12$

Check

$\frac{x}{3} = 4$

$\frac{12}{3} \stackrel{?}{=} 4$

$4 = 4$

4. a) Solve and check:

$\frac{2n}{3} = 4$

4. a) $\frac{2n}{3} = 4$

$\frac{3}{2} \cdot \frac{2n}{3} = \frac{3}{2} \cdot 4$

$n = 6$

Check

$\frac{2n}{3} = 4$

$\frac{2 \cdot 6}{3} \stackrel{?}{=} 4$

$4 = 4$

b) Guide students to realize that, when applying the multiplication or division properties, the reciprocal or multiplicative inverse should be used.

Summary:

1. What is the multiplication property of equalities?
2. What is the division property of equalities?
3. When using the multiplication or division properties to solve an equation, what number may never be used?

SUGGESTED EXERCISES

1. Perform the indicated operations.

a) $(-5)(+9)$

b) $(-8)(-9)$

c) $(-6)(12)$

d) $\frac{-56}{+7}$

e) $\frac{+42}{-14}$

f) $(+9)(15)$

g) $(-15)(-15)$

h) $(100)(-45)$

i) $\frac{-120}{-15}$

j) $\frac{+640}{16}$

2. Solve each equation. Check your solution.

a) $14x = 28$

b) $16n = 128$

c) $8y = 88$

d) $13t = 143$

e) $\frac{x}{4} = 2$

f) $\frac{x}{-2} = -4$

g) $\frac{x}{+5} = -2$

h) $\frac{m}{-8} = 3$

i) $72 = 9x$

j) $5x = -35$

k) $-12m = 60$

l) $-240 = -8y$

m) $\frac{2k}{5} = 14$

n) $-\frac{3}{4}a = 9$

o) $\frac{s}{5} = -5$

p) $-\frac{2p}{4} = -3$

Lesson 8

Aim

To solve equations of the form $ax + b = c$ and $ax - b = c$

Motivation

Mr. Jones is installing copper pipe along two adjacent sides of a rectangular-shaped basement. He notes that twice the width of the basement increased by 13 feet is equal to the 43 feet in the length. How wide is the basement?

Development

Teacher Activity	Student Activity
1. Guide the students to set up an open sentence expressing the relationship of the length and width.	1. Let w = the width in feet $2w + 13 = 43$
2. Permit students to use the different properties of equalities to solve for w. Students will soon discover that the work will be less cumbersome if the addition property is used first.	2. Division Property First $2w + 13 = 43$ $\frac{2w}{2} + \frac{13}{2} = \frac{43}{2}$ $w + 6\frac{1}{2} = 21\frac{1}{2}$ $- 6\frac{1}{2} = -6\frac{1}{2}$ $w = 15$

Addition Property First

$2w + 13 = 43$

$- 13 = -13$

$\frac{2w}{2} = \frac{30}{2}$

$w = 15$

Check

$2w + 13 = 43$

$2 \cdot 15 + 13 = 43$

$30 + 13 = 43$

$43 = 43$

3. Have the students state a rule for solving equations of the form $ax \pm b = c$.

3. When solving equations of the form $ax \pm b = c$,
   a) use the addition property first.
   b) Complete the solution by then using the division or multiplication property.

Summary

When solving the equation $6t - 7 = 41$, which property should be used first?

SUGGESTED EXERCISES

1. Solve and check.

a) $9x + 8 = 80$

b) $8x + 9 = 57$

c) $15a - 39 = 21$

d) $11 = 4a - 13$

e) $10x - 17 = 13$

f) $16t + 3 = 51$

g) $7x + 19 = 68$

h) $4x - 23 = 19$

i) $27 = 8y - 5$

j) $35 = 6y + 13$

2. Solve and check.

a) $3x + 7 = 31$

b) $5r + 6 = 71$

c) $12y + 35 = 59$

d) $11a + 23 = 70$

e) $-9x + 1 = 10$

f) $15j - 44 = 11$

g) $45 = 18x + 45$

h) $7y + 6 = 41$

i) $66 = 7a + 19$

j) $-51 = 16m + 13$

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